# UGC POINT

## LEADING INSTITUE FOR CSIR-JRF/NET,GATE & JAM

BOOKLET CODE

**B**

SUBJECT CODE

**05**

## PHYSICAL SCIENCE

## TEST SEREIS # 4

### MATHEMATICAL, THERMAL, STATISTICAL, SOLID STATE PHYSICS

Date: 2/6/2015

Timing: 2:00 H

Maximum Marks: 80

### Instructions

1. This test paper has a total of 40 questions carrying 80 marks. All Question are compulsory
2. Read the Questions carefully and mark your appropriate response to the OMR sheet
3. There is Negative marking of 1/4 for Each wrong answer
4. Mark the response by **Black** or **Blue** Ball Pen only
5. Any other belongings like Book/ Notes / Electronic device etc are not permitted in the examination hall.
6. Submit your answer sheet (OMR Sheet) to the invigilator before leaving the examination hall

UGC POINT ACADEMY
India's Leading Institute for : CSIR, NET & GATE

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1. The vector perpendicular to the plane of the vectors $2\hat{i}-\hat{j}+4\hat{k}$ and $5\hat{i}+2\hat{j}-2\hat{k}$ is

   (1) $\hat{i}-2\hat{j}-\hat{k}$ (2) $2\hat{i}-8\hat{j}-3\hat{k}$ (3) $-\hat{i}+2\hat{j}+\hat{k}$ (4) $3\hat{i}-2\hat{j}+8\hat{k}$

2. The product of four matrices is given as ABCD. The transpose of this product i.e. $(ABCD)^T = ?$

   (1) $C^TA^TO^TB^T$ (2) $C^TD^TA^TB^T$ (3) $A^TB^TC^TD^T$ (4) $D^TC^TB^TA^T$

3. For $AA^+$ to be Hermitian A is

   (1) Hermitian (2) Unitary (3) Orthogonal (4) A can be any arbitrary matrix

4. If $H$ is Hermitian and $U$ is unitary then $U^{-1}HU$ is

   (1) Hermitian (2) Unitary (3) Orthogonal (4) Anti Hermitian

5. The eigen values of the matrix

   $A=\begin{pmatrix}1 & -4 & 2\\ -4 & 1 & -2\\ 2 & -2 & -2\end{pmatrix}$ Are

   (1) $2,-1,-1$ (2) $4,-2,-2$ (3) $-4,2,2$ (4) $6,-3,-3$

6. The equation of a conic is given as

   $$2x^2+4xy-y^2=24$$

   The equation relative to principal axes is:

   (1) $3x'^2-4y'^2=24$ (2) $3x'^2-2y'^2=24$ (3) $2x'^2-3y'^2=24$ (4) $x'^2-2y'^2=24$

7. The angular momentum of a particle m is $\vec{L}=m\vec{r}\times\left(\frac{d\vec{r}}{dt}\right)$, then $\frac{d\vec{L}}{dt}=?$

   (1) $2m\vec{r}\times\frac{d^2\vec{r}}{dt^2}$ (2) $-m\vec{r}\times\frac{d^2\vec{r}}{dt^2}$ (3) $m\vec{r}\times\frac{d^2\vec{r}}{dt^2}$ (4) $-m\vec{r}\times\frac{d^2r}{dt^2}+m\left(\frac{d\vec{r}}{dt}\times\frac{d\vec{r}}{dt}\right)$

8. Suppose that a hill has the equation $z=32-x^2-4y^2$, where z = height measured from some reference level in (hundreds of feet). If you start at the point (3, 2) and move in the direction $\hat{i}+\hat{j}$, in which direction are you going?

   (1) Up hill (2) down hill (3) neither up nor down (4) none of the above

9. In the previous question, what is the rate at which you are moving

   (1) $11\sqrt{2}$ (2) 11 (3) 0 (4) None of these

10. For the force $\vec{F} = -y\hat{i} + x\hat{j} + z\hat{k}$ the work done in moving a particle from $(1,0,0)$ to $(-1,0,\pi)$ along $x = \cos t,\ y = \sin t,\ z = t$ is

(1) $\pi - \frac{\pi^2}{2}$ (2) $\frac{\pi}{2} + \pi^2$ (3) $\pi + \frac{\pi^2}{2}$ (4) $\frac{\pi}{2} - \pi^2$

11. The summation

$$\frac{1}{2^2-1} + \frac{1}{4^2-1} + \frac{1}{6^2-1} + \ldots\ldots =$$

(1) $\frac{1}{2}$ (2) 1 (3) $\frac{\pi}{2}$ (4) $\pi$

12. The integral $\int_0^\infty \frac{1-\cos\alpha}{\alpha^2} d\alpha$ is

(1) $\frac{\pi}{2}$ (2) $\pi$ (3) $\frac{3\pi}{2}$ (4) $2\pi$

13. The solution of the differential equation $y'' - 5y' + 6y = 2e^x + 6x - 5$ is :

(1) $(A+1)e^x + Be^{2x} + x$ (2) $y = Ae^{3x} + Be^{2x} + e^x + x$

(3) $y = Ae^{3x} + Be^{2x} + x$ (4) $y = Ae^{4x} + Be^{2x} + e^x + x$

14. The integral $\int_0^\infty \phi(x)\delta\left(x^2 - a^2\right)dx$ is

(1) $\frac{\phi(|a|)}{|a|}$ (2) $\phi(|a|)$ (3) $\phi\left(|a|^2\right)$ (4) $\frac{\phi(|a|)}{2|a|}$

15. Choose the correct relation for Bessel function is

(1) $J_2(x) = \left(\frac{2}{x}\right)J_1(x) + J_0(x)$ (2) $J_1(x) - J_3(x) = \left(\frac{4}{x}\right)J_2(x)$

(3) $J_1(x) + J_3(x) = \left(\frac{4}{x}\right)J_2(x)$ (4) $J_2(x) = J_0(x) - \left(\frac{2}{x}\right)J_0(x)$

16. If 4 different letters are put at random into 4 envelops bearing the address of different recipients, what is the probability that at least one letter gets into correct envelop?

(1) $\frac{1}{4}$ (2) $\frac{1}{8}$ (3) $\frac{1}{16}$ (4) $\frac{1}{2}$

17. The integral $\int_0^{\infty} \frac{\cos 2x}{9x^2+4} dx$ is

(1) $\frac{\pi e^{-4/3}}{12}$ (2) $\frac{-\pi e^{-4/3}}{12}$ (3) $\frac{\pi e^{-4/3}}{6}$ (4) $\frac{-\pi e^{-4/3}}{6}$

18. The integral $\int_0^{\infty} \frac{dx}{1+x^2}$ is

(1) $\pi$ (2) $\frac{\pi}{2}$ (3) $\frac{\pi}{4}$ (4) $\frac{3\pi}{4}$

19. The residue of the function $\frac{1}{z}\sin(2z+5)$ at infinity is

(1) $-\sin(5)$ (2) $\sin(5)$ (3) $\sin(2)$ (4) 1

20. If C is a circle of radius $\rho$ about $z_0$, then

(1) $2\pi$ (2) $2\pi i$ (3) $-2\pi i$ (4) 0

21. There are three energy levels 0, E and $2E$ (where E > 0) levels. Canonical partition function of two identical fermions is

(1) $e^{-2\beta E}+e^{-4\beta E}+e^{-3\beta E}$ (2) $1+e^{-\beta E}+e^{-2\beta E}$

(3) $e^{-\beta E}+e^{-2\beta E}+e^{-3\beta E}$ (4) $e^{-\beta E}+e^{-3\beta E}+e^{-5\beta E}$

22. The total energy contained in a box filled with radiation of all frequencies at temperature T is infinite. The value of Phank's constant is

(1) 1 (2) $6.626\times10^{-34}\,J.s$ (3) 0 (4) $\infty$

23. Consider a system of 2 fermions which can occupy any of the 4 available energy states with equal probability. The entropy of the system is

(1) $k_B \ln 6$ (2) $2k_B \ln 2$ (3) $2k_B \ln 4$ (4) $3k_B \ln 4$

24. For a system of 3 bosons, each of which can occupy any of the two energy levels 0 and $\varepsilon$, the mean energy of the system at a temperature T with $\beta = \frac{1}{kT}$ is given by

(1) $\frac{1+\varepsilon e^{-\beta\varepsilon}+2\varepsilon e^{-2\beta\varepsilon}}{1+e^{-\beta\varepsilon}+e^{-2\beta\varepsilon}+e^{-3\beta\varepsilon}}$ (2) $\frac{1+\varepsilon e^{-\beta\varepsilon}+2\varepsilon e^{-2\beta\varepsilon}}{1+e^{-\beta\varepsilon}+e^{-2\beta\varepsilon}+e^{-3\beta\varepsilon}}$

(3) $\frac{\varepsilon e^{-\beta\varepsilon}+2\varepsilon e^{-2\beta\varepsilon}+3\varepsilon e^{-3\beta\varepsilon}}{1+e^{-\beta\varepsilon}+e^{-2\beta\varepsilon}+e^{-3\beta\varepsilon}}$ (4) $\frac{1+\varepsilon e^{-\beta\varepsilon}+3\varepsilon e^{-2\beta\varepsilon}}{1+e^{-\beta\varepsilon}+e^{-2\beta\varepsilon}+e^{-3\beta\varepsilon}}$

25. At a given temperature T, the average energy per particle of a non interacting gas of n-dimensional classical harmonic oscillators is $2k_BT$ , then the value of n is

(1) 1 (2) 2 (3) 3 (4) 4

26. A paramagnetic system consisting of $10^{10}$ spin-half particles is placed in an external magnetic field. It is found that $5\times10^9$ spins are aligned parallel and the remaining spins are aligned anti parallel to the magnetic field. The entropy of the system is:

(1) $10^{10}k_B\ln 2$ (2) $5\times10^9k_B\ln 2$ (3) $1.5\times10^{10}k_B\ln 2$ (4) $2\times10^{10}k_B\ln 2$

27. The total number of accessible states of 100 non-interacting particles of spin $\frac{1}{2}$ is:

(1) $2^{100}$ (2) $100^2$ (3) $2^{50}$ (4) 100

28. A heat pump working on a Carnot cycle maintains the inside temperature of a house at $22^oC$ by supplying $450kJ/s$. The heat taken is $417kJ/s$. The outside temperature is

(1) $0^oC$ (2) $30^oC$ (3) $300\,k$ (4) $275\,k$

29. At low temperature the graph between $\frac{C_V}{T}$ and $T^2$ (where $C_V$ is the specific heat of metal) is

(1) Parabola (2) hyperbola (3) straight line (4) none of these

30. Consider black body radiation in a cavity maintained at 2000 K. If the volume of the cavity is reversibly and adiabatically increased from 10 $cm^3$ to V, so that the temperature becomes 500K, the value of V is

(1) $100\,cm^3$ (2) $640\,cm^3$ (3) $64\,cm^3$ (4) $1000\,cm^3$

31. The entropy S and pressure of a photon gas are

$S=\frac{4}{3}aVT^3,\ P=\frac{a}{3}T^4$

Where $a$ is constant the free energy F of the gas is

(1) $-\frac{a}{3}TV^4$ (2) $\frac{a}{3}TV^4$ (3) $\frac{a}{3}VT^4$ (4) $-\left(\frac{a}{3}\right)VT^4$

32. At what temperature of a nitrogen and oxygen mixture do the most probable velocities of nitrogen and oxygen molecules differ by $30\,m/s$ ?

(1) 370 K (2) 500 K (3) 400 K (4) 700 K

33. A Carnot's engine whose temperature of source is 400 K takes 500 colories of heat at this temperature and rejects 400 calories of heat to the sink. The temperature of the sink is
(1) 300 k (2) 220 k (3) 320 k (4) 250 k

34. For a three dimensional crystal having N primitive unit cells with a basis of 2 atoms, the number of optical branches is
(1) $3N-6$ (2) 6 (3) 2 (4) 3

35. Consider X-ray diffraction from a crystal with a body centered cubic (bcc) lattice. The lattice plane for which there is no diffraction peak is
(1) $(2,1,2)$ (2) $(1,1,1)$ (3) $(2,0,0)$ (4) $(3,1,1)$

36. Metallic monovalent sodium crystallizes in bcc structure. If the concentration of conduction electrons is $3.125\times10^{22}cm^{-3}$ the side of the unit cell is
(1) $4\times10^{-8}cm$ (2) $4\times10^{-9}cm$ (3) $4\times10^{-7}cm$ (4) $2\times10^{-9}cm$

37. When X-ray of wave length 0.03 nm is used to analyze $CaCO_3$ crystal, the smallest angle of Bragg scattering is $2.9^o$. The distance between adjacent lattice planes is
(1) 0.03 nm (2) 0.3 nm (3) 3 nm (4) 0.15 nm

38. The kinetic energy of a free electron at the midpoint of a side of the first Brillouin zone of a two dimensional square lattice is smaller than that of an electron at a corner of the zone by a factor b. the value of b is
(1) $\frac{1}{4}$ (2) $\frac{1}{2}$ (3) $\frac{1}{6}$ (4) $\frac{1}{8}$

39. In a powered diffraction pattern recorded from a Fcc sample using X-rays, the fist peak appears at $30^o$. The third peak will appear at
(1) $32.8^o$ (2) $33.7^o$ (3) $34.8^o$ (4) $35.3^o$

40. The value of plasma frequency of a free electron gas in $(Hz)$ changes from $5.7\times10^{15}$ to $5.7\times10^{14}$, the number density changes from $10^{28}$ to
(1) $10^{23}$ (2) $10^{24}$ (3) $10^{25}$ (4) $10^{26}$

# UGC POINT

## Test Series # 4
## (Physics)
## Answer Key

| | | | |
|---|---|---|---|
| 1.(2) | 11.(1) | 21.(3) | 31.(4) |
| 2.(4) | 12.(1) | 22.(3) | 32.* |
| 3.(4) | 13.(2) | 23.(1) | 33.(3) |
| 4.(1) | 14.(4) | 24.(3) | 34.(4) |
| 5.(4) | 15.(3) | 25.(2) | 35.(2) |
| 6.(2) | 16.Probability 5/8 | 26.(2) | 36.(1) |
| 7.(3) | 17.(1) | 27.(1) | 37.(2) |
| 8.(2) | 18.(2) | 28.(1) | 38.(2) |
| 9.(1) | 19.(1) | 29.(3) | 39.(4) |
| 10.(3) | 20.(4) | 30.(2) | 40.(4) |

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