SOLUTIONS (TEST SERIES 1) PH

1. (2)

2. (1) for compound pendulum

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{mgl}}$$

where $\omega = \sqrt{\frac{mgl}{I}}$

If $k \rightarrow$ radius of hydration, then

$$I = mk^2 + ml^2$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{mk^2+ml^2}{mgl}}$$

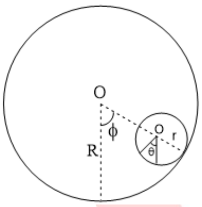
or

$$\boxed{T = 2\pi\sqrt{\frac{k^2+l^2}{gl}}}$$

3. (1) Since the cylinder rolls without slipping equation of constraints

$$r\theta = (R-r)\phi$$

or $r\dot{\theta} = (R-r)\dot{\phi}$ ……(i)



K.E. = due to translation motion + due to rotational motion

$$= \frac{1}{2}m[(R-r)\dot{\phi}]^2 + \frac{1}{2}I_0\dot{\theta}^2$$

$$= \frac{3}{4}m(R-r)^2\dot{\phi}^2 \left[since\ I_0 = \frac{1}{2}mr^2\right]$$

Potential energy

$$V = -mg(R-r)cos\phi$$

and $V = 0$ for $\phi = 90^o$

Lagrangian,

$$L = T - V$$

$$= \frac{3}{4}m(R-r)^2\dot{\phi}^2 + mg(R-r)cos\phi \ldots\text{(ii)}$$

Equation of motion,

$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\phi}}\right) - \frac{\partial L}{\partial\phi} = 0$$

Using equation (ii)

$$\frac{3}{2}(R-r)\ddot{\phi} + g\phi = 0$$

[for small oscillations $sin\phi = \phi$]

or $\ddot{\phi} + \frac{2g}{3(R-r)}\phi = 0$ …..(iii)

compare with equation

$$\ddot{\phi} + \omega^2\phi = 0$$

So ω, the frequency of small oscillations

$$\omega = \sqrt{\frac{2g}{3(R-r)}}$$

The period of oscillation

$$\boxed{T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{3(R-r)}{2g}}}$$

4. (1)
5. (2) The eastward derivation of the falling bodies is $y' = \frac{1}{3}\omega g t^3 cos\phi$

Time taken by the body in going up to the point it will come at rest is $\left(\frac{2h}{g}\right)^{1/2}$.

Similar time will be taken by this body in coming down to the surface of the earth.

Total time of flight = $2\left(\frac{2h}{g}\right)^{1/2}$

$$\therefore y' = \frac{1}{3}\omega g\left[2\left(\frac{2h}{g}\right)^{1/2}\right]^3 cos\phi\left(\frac{2h}{g}\right)^{1/2}$$

$$\boxed{y' = \frac{16}{3}\omega h\, cos\phi\left(\frac{2h}{g}\right)^{1/2}}$$

6. (4)
7. (2) Condition for canonical transformation

[Q,P] = 1 ……..(1)

where $[Q, P] = \frac{\partial Q}{\partial q_i}\frac{\partial P}{\partial p_i} - \frac{\partial Q}{\partial p_i}\frac{\partial P}{\partial q_i}$ …..(2)

$$Q = q^m cos\ np;$$

$$P = q^m \sin np$$

$$\left.\frac{\partial Q}{\partial q_i} = mq^{m-1}cosnp; \frac{\partial P}{\partial q_i} = mq^{m-1}sinnp\right\}$$

$$\left.\frac{\partial Q}{\partial p_i} = -q^m \cdot nsinnp; \frac{\partial P}{\partial p_i} = nq^m cosnp\right\}$$

….(3)

Using equation (3) in (1) and (2)

$$m \cdot q^{m-1}cosnp \cdot nq^m cosnp + q^m \cdot n\, sinnp = 1$$

$$mq^{m-1}\ sinnp = 1$$

$$\Rightarrow mnq^{2m-1}(cos^2np + sin^2np) = 1$$

$$mnq^{2m-1} = 1$$

Thereby giving

$$mn = 1$$

and $2m - 1 = 0$

gives $\boxed{m = \frac{1}{2}, n = 2}$

8. (1) $Q = \log(1 + q^{1/2}cosp)$

we get,

$$e^Q = 1 + q^{1/2}cosp$$

or $q = (e^Q - 1)^2 sec^2 p$

Now, we know

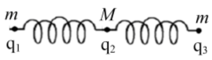
$$\frac{\partial F_3}{\partial p} = -q = -(e^Q - 1)^2 sec^2 p$$
$$F_3 = \int -(e^Q - 1)^2 sec^2\, p dp$$
Choosing constant of integration as zero
$$\boxed{F_3 = -(e^Q - 1)^2 tanp}$$

9. (4)
10. (1)
11. (4) For angular frequency, we have to find out equation of motion.
We use Taylor's series expansion
$V(a + x) = V(a) + V'(a)x + \frac{1}{2}V"(a)x^2 + \cdots$

$V(a) = kma^3$
$V(r) = kmr^3$
$\Longrightarrow V(a + x) = kma^3 + 3.kma^2 + \frac{1}{2}\cdot 6kma \cdot x^2 + \cdots$
So, equation of motion
$$m\ddot{x} + \left[\frac{3V'(a)}{a} + V"(a)\right]x = 0$$
$$\Longrightarrow m\ddot{x} + \left[\frac{3.3kma^2}{a} + 6kma\right]x = 0$$
$$\Longrightarrow \quad \ddot{x} + 15ax = 0$$
Compare with $\quad \ddot{x} + \omega^2 x = 0$
We get angular frequency $\boxed{\omega = \sqrt{15ka}}$

12. (3) $\quad r = e^{-\theta}$
$$\Longrightarrow \quad u = \frac{1}{r} = e^{\theta};\ \frac{d^2u}{d\theta^2} = e^{\theta}$$
Equation of orbit
$$\frac{d^2u}{d\theta^2} + u = -\frac{m}{l^2u^2} f\left(\frac{1}{u}\right)$$
$$e^{\theta} + e^{\theta} = -\frac{m}{l^2u^2} f\left(\frac{1}{u}\right)$$
$$\Longrightarrow \quad f\left(\frac{1}{u}\right) = -2.e^{\theta}.\frac{l^2u^2}{m}$$
$$f\left(\frac{1}{u}\right) = -\frac{2l^2}{m}u.u^2$$
or $\quad \boxed{f(r) \propto \frac{1}{r^2}}$

13. (3)
14. (2)
15. (1)
16. (3)



All three atoms are on one straight line. Two masses m are symmetrically located on each side of an atom of mass M. There exists an elastic bond between central atom and the end atoms, of force constant k.
$$T = \frac{1}{2}m(\dot{q}_1^2 + \dot{q}_3^2) + \frac{1}{2}M\dot{q}_2^2$$
So, that the T matrix is diagonal
$$T = \begin{pmatrix} m & 0 & 0 \\ 0 & M & 0 \\ 0 & 0 & m \end{pmatrix} \quad \ldots\ldots (1)$$
Potential energy
$$V = \frac{1}{2}k(q_2 - q_2)^2 + \frac{1}{2}\,k(q_3 - q_2)^2$$
$$= \frac{1}{2}k(q_1^2 + 2q_2^2 + q_3^2 - 2q_1q_2 - 2q_2q_3)$$
Hence, the V matrix has the form
$$V = \begin{pmatrix} k & -k & 0 \\ -k & 2k & -k \\ 0 & -k & k \end{pmatrix} \quad \ldots\ldots (2)$$
Combing these two matrices, the secular equation appears as
$$|V - \omega^2 T| = \begin{vmatrix} k - \omega^2 m & -k & 0 \\ -k & 2k - \omega^2 M & -k \\ 0 & -k & k - \omega^2 m \end{vmatrix} =$$
direct evaluation of the determinant leads to the cubic equation in ω^2
$$\omega^2(k - \omega^2 m)(k(M + 2m) - \omega^2 Mm) = 0 \quad \ldots. (3)$$
with solution
$$\omega_1 = 0$$
$$\omega_2 = \sqrt{\frac{k}{m}}$$
$$\omega_3 = \sqrt{\frac{k}{m}\left(1 + \frac{2m}{M}\right)}$$
$\omega_1 = 0 \rightarrow$ refers no oscillation
$\omega_2 \rightarrow$ Shows S.H.M.
$\Longrightarrow$ So normal frequency for triatomic is
$$\boxed{\omega_3 = \sqrt{\frac{k}{m}\left(1 + \frac{2m}{M}\right)}}$$

17. (1)
18. (2)
19. (3)
20. (4)
21. (2) $\quad L = \frac{1}{2}\dot{x}^2 - \frac{1}{2}\omega^2x^2 - \alpha x^3 + \beta x\dot{x}^2$
$$p_x = \frac{\partial L}{\partial \dot{x}} = \dot{x} + 2\beta x\dot{x}$$
or $\quad \dot{x} = \frac{p_x}{1+2\beta x}$
The Hamiltonian
$$H = p_x\dot{x} - L$$
$$= \dot{x}^2 + 2\beta x\dot{x}^2 - \frac{1}{2}\dot{x}^2 + \frac{1}{2}\omega^2x^2 + \alpha x^3 - \beta x\dot{x}^2$$
$$= \frac{1}{2}\dot{x}^2 + \beta x\dot{x}^2 - \frac{1}{2}\omega^2x^2 + \alpha x^3$$
$$= \frac{\dot{x}^2(1+2\beta x)}{2} + \frac{1}{2}\omega^2x^2 + \alpha x^3$$

22. (2) K.E. of the particle is not conserved infact total energy of the particle is always conserved.

23. (1) angular momentum is conserved during revolution of planet, because gravitational force is a central force.

$$K.E. = \frac{1}{2}mv^2$$

or $$= \frac{m^2v^2r^2}{2mr^2}$$

$$= \frac{L}{2mr^2}$$

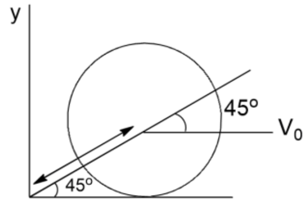
or since L is constant hence

$$K.E \propto \frac{1}{r^2}$$

$$\frac{E_2}{E_1} = \frac{r_1^2}{r_2^2} = \frac{1}{16}$$

1: 16

24. (3)



$$\vec{L} = \vec{L}CM + M\,\bar{r}_0 \times \bar{v}_0$$

$$= I\omega + Mr_0v_0 \sin 45^\circ$$

$$= Ma^2\omega + M\sqrt{2}a.\omega a\frac{1}{\sqrt{2}}$$

$$= 2Ma^2\omega$$

25. (2)

26. (4) Coriolis force $= 2m\vec{\omega} \times \vec{v}$

$$= 2m\omega v sin\theta$$

Since the axis of rotation is perpendicular to the wire $\theta = 90^o$

27. (4) The xy component of the moment of inertia tensor

$$I_{xy} = -\Sigma mxy = I_{yx}$$

two particles of mass m located at $(-1,1)$ and $(1,-1)$ other two particles of mass $2m$ are located at $(1,1)$ and $(-1,-1)$

$$I_{xy} = -[m_1x_1y_1 + m_2x_2y_2 + m_3x_3y_3 + m_4x_4y_4]$$

$$= -[m\times-1\times1 + m\times1\times-1 + 2m\times1\times1 + 2m\times1\times-1]$$

$$= -[-m - m + 2m + 2m]$$

$$= -[2m]$$

28. (2)

29. (2)

30. (3)

31. (2) $E = mc^2 - m_0c^2$

$$= \frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}} - m_0c^2$$

$$9.69 = \frac{0.51}{\sqrt{1-\frac{v^2}{c^2}}} - 0.51$$

$$\Rightarrow \quad 10.2 = \frac{0.51}{\sqrt{1-\frac{v^2}{c^2}}}$$

or $$\sqrt{1-\frac{v^2}{c^2}} = \frac{0.51}{10.2} = 0.05$$

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\Rightarrow \quad \frac{m}{m_0} = \frac{1}{0.05} = \frac{100}{5}$$

or $$\frac{m}{m_0} = \frac{20}{1}$$

32. For the stokes line of rotational Raman spectra

$6B = 12.96$

$$\boxed{B = 2.16 cm^{-1}}$$

33. (3)

34. An interaction between the internal magnetic field produced by the orbital angular momentum of the electrons in an atom and the spin magnetic dipole moment of the nucleus causes a hyperfine splitting of the spectral terms

35. $$B\alpha\frac{1}{I}\alpha\frac{1}{\mu}$$

$$\therefore\ B_1\alpha\frac{1}{I_1},\ B_2\alpha\frac{1}{I_2}$$

$$\frac{B_1}{B_2} = \frac{I_2}{I_1}$$

$$\frac{I_2}{I_1} = \frac{3.842}{3.673}$$

$$\boxed{\frac{I_2}{I_1} = 1.046}$$

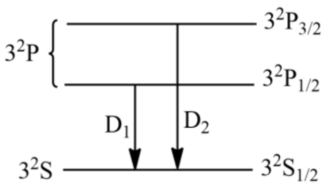
36. isotopes have same atomic number. So, they have same chemical property, but have different atomic weight. So, they have different rotational spectra.

37. Doppler breadth $\delta\lambda = 1.67\frac{\lambda_0}{c}\sqrt{\frac{2RT}{\mu}}$

$$\boxed{\delta\lambda \alpha T^{1/2}}$$

Doppler broadening depending on temperature for more doppler broadening temperature will be more and less broadening mass will be more

38. For the D_1 line, the possible transition phase is, ${}^2P_{1/2} \rightarrow {}^2S_{1/2}$ and for D_2 line the possible transition phase is ${}^2P_{3/2} \rightarrow {}^2S_{1/2}$.



The ratio of intensities of the D_1 and D_2 lines, $= \frac{D_1}{D_2} = \frac{2J_1+1}{2J_2+1}$

$$\frac{D_1}{D_2} = \frac{2\times\frac{1}{2}+1}{2\times\frac{3}{2}+1} = \frac{2}{4}$$

$$\frac{D_1}{D_2} = \frac{1}{2}$$

39. For ${}^3P$ spectroscopic term

Multiplicity $= (2S+1) = 3$

Hence total angular momentum

$J = |L-S| \ldots\ldots\ldots. |L+S|$

$= 0, 1, 2$

So, total representation of that spectroscopic term is ${}^3P_{0,1,2}$. Degeneracy (2J+1) for individual term are

For $J = 0 \Rightarrow 1$

$J = 1 \Rightarrow 3$

$J = 2 \Rightarrow 5$

40. Given electronic configuration $2s^1 3d^1$

$|L-S| \le J \le |L+S|$

For $2S^1, L_1 = 0, S_1 = \frac{1}{2} \Rightarrow J_1 = \frac{1}{2}$

For $3d^1, L_2 = 2, S_2 = \frac{1}{2} \Rightarrow J_2 = \frac{3}{2}, \frac{5}{2}$

Now $|J_1 - J_2| \le J \le |J_1 + J_2|$

For $J_1 = \frac{1}{2}, J_2 = \frac{3}{2} \Rightarrow J = 1, 2$

For $J_1 = \frac{1}{2}, J_2 = \frac{5}{2} \Rightarrow J = 2, 3$

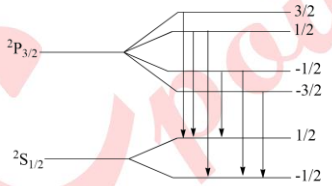
$\therefore \{J_1, J_2\}_J = \left\{\frac{1}{2}, \frac{3}{2}\right\}_{1,2}$ and $\left\{\frac{1}{2}, \frac{5}{2}\right\}_{2,3}$

41. $2\omega e - 6\omega_e x_e = v^I_{overtone}(cm^{-1})$

$2 \times 2990 - 6 \times 2990 \times x_e = 1.96$

$$\boxed{x_e = 0.02}$$

42. In presence of external magnetic field (weak-field) Zeeman Effect splits ${}^2S_{1/2}$ into 2 state $\left(m_J = \frac{1}{2}, -\frac{1}{2}\right)$ and ${}^2P_{3/2}$ level into 4 state $\left(m_J = \frac{3}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}\right)$



43. $v = 1000(2n-1)$

$= 1000, 3000, 5000, \ldots. cm^{-1}$,

for $n = 1,2,3$ ... and

$v = -1000(2n+1)$

$= 1000, 3000, 5000 \ldots. cm^{-1}$

for $n = -1, -2, -3$ ....

This means that the separation between any two consecutive lines is $2000 cm^{-1}$; so that

$2B = 2000 cm^{-1}$

$B = 1000 cm^{-1} = 10^5 m^{-1}$

The moment of inertia of the molecule is therefore

$$I = \frac{h}{8\pi^2 Bc}$$

$$= \frac{6.63 \times 10^{-34} Js}{8 \times (3.14)^2 \times 10^5 m^{-1} \times (3 \times 10^8 ms^{-1})}$$

$= 2.8 \times 10^{-49} kgm^2$

44. (2)

45. Wave length of the first line of the Lyman series is given by

$\frac{1}{\lambda} = R_M\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3}{4}R_M$ or

$\lambda = \frac{4}{3R_M}$; Where $R_M = \frac{\mu}{m_e} R_\infty$

Reduced mass,

$\mu = \frac{200m_e \times 1836m_e}{200m_e + 1836m_e} = 180.35m_e$

So $R_M = 180.35R_\infty$

$\lambda = \frac{4}{3R_M} = \frac{4}{3\times 180.35R_\infty} = \frac{4}{3\times 180.35\times 103737}$

$= 6.67\times 10^{-8} cm = 6.67A^o$

46. (1)

47. (2)

48. ….

49. The splitting of sodium yellow line into two components (D_1 and D_2) arises due to splitting of 3P level. The wave number splitting of 3P level.

The wave number splitting of 3P level is

$\Delta\nu = (2.1\times 10^{-3} eV)\times(8066 cm^{-1}/eV) = 16.9cm^{-1}$

The mean wave-length of the sodium yellow line is $\lambda = 5893A^o = 5893\times 10^{-8} cm$

Now $\nu = \frac{1}{\lambda}$

$\Delta\nu = -\frac{1}{\lambda^2}\Delta\lambda$

$\nu = -\frac{1}{\lambda^2}\Delta\lambda$

$|\Delta\lambda| = \Delta\nu\times\lambda^2$

$= 16.9cm^{-1}\times(5893\times 10^{-8} cm)^2$

$= 5.87\times 10^{-8} cm$

$= 5.87A^o$

50. The first member of principle series correspond to transmission $3P-3S$. The energy corresponding to its wave-length of $5893A^o$ is

$E_1(3P-3S) = \frac{hc}{\lambda} = \frac{(6.63\times 10^{-34} J.S)(3.0\times 10^8 ms^{-1})}{5893\times 10^{-10} m}$

$= 1.375\times 10^{-19} J$

$= \frac{3.375\times 10^{-19} J}{1.67\times 10^{-19} J/eV} = 2.11eV$

The energy corresponding to the first excited state 4s of sodium relative to the ground state 3S is

$E_2(4S-3S) = 3.18eV$ (given)

The first member of the sharp series corresponds to the transition 4S-3P. The corresponding energy is

$E(4S-3P) = E_2(4S-3S) - E_1(3P-3S)$

$= 3.18eV - 2.11eV = 1.01eV$

The corresponding wave length is

$\lambda = \frac{hc}{E} = \frac{(6.63\times 10^{-34} J.s)(3\times 10^8 ms^{-1})}{(1.07eV)(1.60\times 10^{-19} J/eV)}$

$\lambda = 11.618\times 10^{-7} m = 11618A^o$

51. The wave number separation between the components of a normal Zeeman is given by

$\Delta\nu = \frac{eB}{4\pi mc} = \frac{(e/m)B}{4\pi c}$

Putting the given values

$\Delta\nu = \frac{(1.76\times 10^{11} C/kg)(0.3N/A.m)}{4\times 3.14\times(3\times 10^8 m/\sec)}$

$= 14m^{-1}$

$\nu\lambda = 1$

$\nu\Delta\lambda + \lambda\Delta\nu = 0$

$|\Delta\lambda| = \frac{\lambda.\Delta\nu}{\nu} = \lambda^2\Delta\nu$

$= (4500\times 10^{-10} m)^2 (14.0m^{-1})$

$= 283.5\times 10^{-14} m$

$= 0.02835\times 10^{-10} m$

$= 0.02835A^o$

52. Doppler half intensity breadth in terms of wave length $\delta\lambda = 1.66\frac{\lambda_0}{c}\sqrt{\frac{2RT}{\mu}}$

R = universal gas constant

T = temperature in Kelvin

μ = atomic mass

$\delta\lambda = 1.66\frac{(5893\times 10^{-10} m)}{3\times 10^8 m/\sec}$

$= \sqrt{\frac{2\times(8.31J/mol-k)\times 500K}{22.99\times 10^{-3} Kg/mol}}$

$= 1.96\times 10^{-12} m$

$= 0.0196\times 10^{-10} m$

$\delta\lambda = 0.0196A^o \simeq 0.02A^o$

53. When electron accelerated through a potential V strike a target, the maximum frequency $V_{\max}$ (or minimum wavelength $\lambda_{\min}$) of the emitted photon is

given by $eV = h\nu_{\max} = h\frac{c}{\lambda_{\min}}$

the minimum voltage for $0.1A^o$ X-ray photon is

$$V = \frac{hc}{e\lambda_{\min}}$$

$$= \frac{\left(6.63\times10^{-34}Js\right)\left(3\times10^{8}ms^{-1}\right)}{\left(1.6\times10^{-19}c\right)\left(0.1\times10^{-10}m\right)}$$

$$= 1.24\times10^{5}J/c = 1.25\times10^{5}V$$

54. From Moseley's law for K_α line, we have

$$\frac{1}{\lambda}\alpha(2-1)^2$$

$$\therefore \quad \frac{\lambda_{Cu}}{\lambda_{Mo}} = \frac{(Z_{Mo}-1)^2}{(Z_{Cu}-1)^2} = \frac{(41)^2}{(28)^2}$$

$$\lambda_{Mo} = 0.71A^o$$

$$\therefore \quad \lambda_{Cu} = \left(0.71A^o\right)\times\frac{(41)^2}{(28)^2} = 1.52A^o$$

55. The wave number of the radiation observed in a rotational transition from J to J+1 is given by

$$\nu = 2B(J+1)$$

where J refers to the lower state for a transition from J = 0 to J = 1, we have

$$\nu = 2B$$

but $\nu = 20.68cm^{-1}$ (given)

$\therefore\ 2B = 20.68cm^{-1}$

$B = 10.34cm^{-1}$

again, the wave number of the radiation observed in the transition $J = 15 \leftarrow 14$ is given by

$\nu = 2B(J+1)$ where J refers to lower state

$= 2B(14+1)$

$= 20.68cm^{-1}\times15 = 310.2cm^{-1}$

The corresponding wave length is

$$\lambda = \frac{1}{\nu} = \frac{1}{310.2cm^{-1}} = 32\times10^{-4}cm = 32\mu$$

56. The frequency of vibration of CO molecule is

$$\nu_{osc} = \frac{1}{2\pi}\sqrt{\frac{K}{\mu}}$$

$$= \frac{1}{2\times3.14}\sqrt{\frac{1870Nm^{-1}}{1.14\times10^{-26}Kg}}$$

$$= 6.45\times10^{13}s^{-1}$$

The vibrational energy of diatomic molecule is

$$E_v = h\nu_{osc}\left(v+\frac{1}{2}\right), \qquad v = 0,1,2......$$

The lowest level correspond to $v = 0$ sts energy is

$$E_{v=0} = \frac{1}{2}h\nu_{osc}$$

$$= \frac{1}{2}\left(6.63\times10^{-34}J.s\right)\left(6.45\times10^{13}s^{-1}\right)$$

$$= 214\times10^{-21}J$$

$$= \frac{21.4\times10^{-21}J}{1.60\times10^{-19}J/eV}$$

$$E_{v=0} = 0.134eV$$

57. As above, the frequency of vibration of CO molecule is

$$\nu_{osc} = 6.45\times10^{13}s^{-1}$$

separation between two successive vibrational energy level is.

$$\Delta E = E_{v+1} - E_v$$

$$= h\nu_{osc}\left(v+\frac{3}{2}\right) - h\nu_{osc}\left(v+\frac{1}{2}\right)$$

$$= h\nu_{osc}$$

$$= \left(6.63\times10^{-34}J.s\right)\left(6.45\times10^{13}s^{-1}\right)$$

$$= 42.8\times10^{-21}J$$

$$= \frac{42.8\times10^{-21}J}{1.60\times10^{-19}J/eV}$$

$$\Delta E = 0.2675eV$$

58. ….

59. (1)

60. ….