SOLUTIONS (TEST SERIES 2) PH

1. (2) In the region where there exists a distribution of charge of volume charge density (i.e. charge per unit volume) ρ, the differential form of Gauss's law is

$$div\ \hat{E} = 4\pi\rho$$

[in CGS Gaussian system]

We have $E = -grad\ V$

$\therefore \quad div(-grad\ V) = 4\pi\rho$

$$-\overline{\nabla}\cdot\nabla V = 4\pi\rho$$

or $\boxed{\nabla^2 V = -4\pi\rho}$

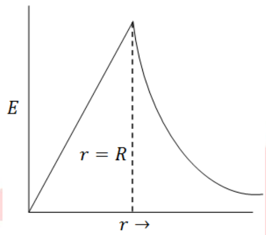
2. (3) The electric field strength due a non-conducting uniformly charged sphere of radius, when point lies outside the charge distribution i.e., $r > R$, varies inversely with r

i.e., $E \propto r^{-1}$

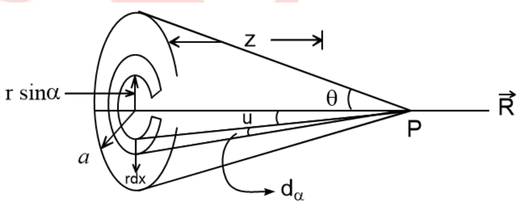
and when point lies inside the charge distribution i.e., $r < R$

$$E \propto r$$

Hence plot will be



3. (1) Magnetic scalar potential for a point on the Z-axis of a circular loop of radius a



As $d\Omega = \frac{ds}{r^2}$

For this case

$$\Omega = \int \frac{ds}{r^2} = \int_0^{\theta} \frac{(2\pi r\ sin\alpha)rd\alpha}{r^2}$$

$$= 2\pi[-cos\theta]_0^{\theta}$$

$$= 2\pi(1 - cos\theta)$$

i.e, $\Omega = 2\left[1 - \frac{Z}{(a^2+Z^2)^{1/2}}\right]$

as $cos\theta = \frac{Z}{(a^2+Z^2)^{1/2}}$

so magnetic scalar potential

$$\boxed{\Phi_m = \frac{\mu_0 I}{4\pi}\Omega = \frac{\mu_0 I}{4\pi}2\pi\left[1 - \frac{Z}{(a^2+Z^2)^{1/2}}\right]}$$

4. (4) $E_z = acos\omega\ x\ cos\omega\ ct$

$H_y = -asin\omega\ x\ sin\omega\ ct$

As $\vec{S} = \vec{E} \times \vec{H}$

i.e., $S = a\cos\omega x \sin\omega x \cos\omega\ ct \sin\omega ct$

$$= -a^2 \cos\omega x \sin\omega x \cos\omega\ ct \sin\omega ct$$

$$= -a^2 \frac{\sin 2\omega x}{2}\cdot\frac{sin2\ \omega ct}{2}$$

$$\boxed{S = -\frac{1}{4}a^2 \sin 2\omega x \sin 2\omega ct}$$

5. (1) we have

$$\frac{dP}{d\Omega} = \frac{1}{4\pi\epsilon_0}\frac{e^2a^2}{4\pi c^3}sin^2\theta$$

The total radiated power is obtained by integrating this equation

$$P = \int_{4\pi}\frac{dP}{d\Omega}d\Omega$$

$$= \int_{4\pi}\frac{1}{4\pi\epsilon_0}\frac{e^2a^2}{4\pi c^3}sin^2\theta d\Omega$$

where, $d\Omega = \frac{(R\sin\theta\ d\phi)(R\ d\theta)}{R^2}$

$$= \frac{1}{4\pi\in_0}\frac{e^2a^2}{4\pi c^2}\int_0^{\pi}\int_0^{2\pi}(\sin^2\theta)\sin\theta\, d\theta\, d\phi$$

$$= \frac{1}{4\pi\in_0}\frac{e^2a^2}{4\pi c^3}\times 2\pi\int_0^{\pi}\sin^3\theta\, d\theta$$

$$= \frac{1}{4\pi\in_0}\frac{e^2a^2}{4\pi c^3}\times 2\pi\times\frac{4}{3}$$

$$\boxed{P = \frac{1}{4\pi\in_0}\frac{2}{3}\frac{e^2a^2}{c^3}}$$

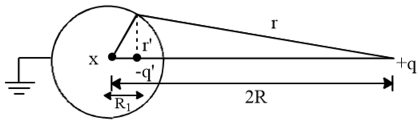
6. (1) $W = -\int_{r_1}^{2r_1} Q\bar{E}.d\bar{r}$

$$= -\int_{r_1}^{2r_1} Q.\frac{k}{r}dr = -[Qk\ln r]_{r_1}^{2r_1}$$

$$= -Qk\left[\ln(2r_1) - \ln r_1\right]$$

$$= -Qk\ln 2$$

7. (1) Applying boundary condition for grounded conducting sphere



$$\frac{q}{4\pi\in_0 r} = -\frac{q'}{4\pi\in_0 r'}$$

or $\frac{q}{r} = -\frac{q'}{r'}$

$$\Rightarrow \quad \frac{q}{R^2 + (2R)^2 - 4R^2\cos\theta}$$

$$= \frac{-q'}{R^2 + R_1^2 - 2R_1 R\cos\theta}$$

So condition requires that

$$R^2 = 2RR_1$$

or $\quad R_1 = \frac{R}{2}$

and $\quad q' = -q\sqrt{\frac{R/2}{2R}}$

$$= -q\sqrt{\frac{1}{4}}$$

$$= -\frac{q}{2}$$

8. (2) $E = \frac{q}{4\pi \in_0 r^2}$

Since $q_1 \to Q, q_2 \to 2Q$

$$r_1 \to r, r_2 \to \frac{r}{2}$$

$$\therefore \frac{E_1}{E_2} = \frac{\dfrac{Q}{4\pi \in_0 r^2}}{\dfrac{2Q}{4\pi \in_0 \left(\frac{r}{2}\right)^2}} = \frac{Q}{r^2} \times \frac{\left(\frac{r}{2}\right)^2}{2Q}$$

$$\frac{E_1}{E_2} = \frac{1}{8}$$

or $\quad E_2 = 8E_1$

9. (1) $V = -E_0\left(1 - \frac{R^3}{r^3}\right) r\cos\theta$

The charge density is related to $\bar{E}$ by relation

$$\bar{E} = \frac{\sigma}{\in_0}$$

or $\quad \sigma = \in_0 \bar{E}$

since $\quad \bar{E} = -\nabla V$

$$\sigma = -\in_0 \left.\frac{\partial V}{\partial r}\right|_{at\, r=R}$$

$$= -\in_0 \left[-E_0\cos\theta - \frac{2R^3}{r^3}\cos\theta E_0\right]_{r=R}$$

$$= -\in_0 \left[-E_0\cos\theta - 2E_0\cos\theta\right]$$

$$\boxed{\sigma = 3\in_0 E_0 \cos\theta}$$

10. (1)

11. (3) The work done in bringing the charge from $x = \infty$ to d is

$$W = \int_{x=\infty}^{d} \vec{F}.dr$$

$$= -\int_{x=\infty}^{\infty} \left(-\frac{q^2}{4\pi \in_0 (2x)^2}\hat{i}\right)$$

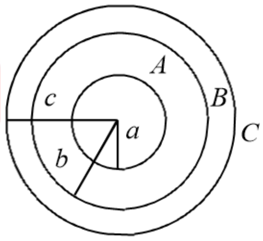
$$= +\frac{q^2}{16\pi \in_0} \int_{x=\infty}^{d} \frac{1}{x^2} dx$$

$$= -\frac{q^2}{16\pi \in_0 d}$$

$$= -\frac{q^2}{4\pi \in_0 (4d)}$$

12. (3) Potential at C

V_c = pot. due to σ on A + pot. due to $-\sigma$ on B + pot. due to σ on C



$$= \frac{1}{4\pi \in_0}\left(\frac{4\pi a^2\sigma}{c}\right) + \frac{1}{4\pi \in_0}\left(\frac{-4\pi b^2\sigma}{c}\right)$$

$$+ \frac{1}{4\pi \in_0}\left(\frac{4\pi c^2\sigma}{c}\right)$$

$$= \frac{\sigma}{\in_0}\left(\frac{a^2}{c} - \frac{b^2}{c} + c\right)$$

13. (2) using Laplace equation

$$\frac{\partial}{dr}\left(r^2\frac{\partial V}{\partial r}\right) = 0$$

$$\Rightarrow r^2\frac{dV}{\partial r} = A$$

i.e. $\quad V = -\frac{A}{r} + B \quad$ ....(i)

but $\quad V = V_a$ at $r = a$

i.e. $\quad V_a = -\frac{A}{a} + B$

and $\quad V = V_b$ at $r = b$

i.e. $\quad V_b = -\frac{A}{b} + B \quad$ ......(iii) eliminating A and B from equation (ii) and (iii) and substitute in equation (i), we get

$$V=\frac{ab(V_a-V_b)}{b-a}\cdot\frac{1}{r}+\frac{bV_b-aV_a}{b-a}$$

Now V_a is at V_0 and $V_b=0$ (according to problem)

$$\therefore \quad V=\frac{abV_0}{b-a}\cdot\frac{1}{r}-\frac{aV_0}{b-a}$$

or $$V=\frac{aV_0}{b-a}\left[\frac{b}{r}-1\right]$$

14. (2) $\sigma=\vec{p}.\hat{n}=P_0\hat{k}.\hat{n}$

$\therefore$ $\boxed{\sigma=p_0}$

and volume charge density

$$\rho=-\bar{\nabla}.\bar{P}$$

$$=-\bar{\nabla}.\left(p_0\hat{k}\right)$$

$\therefore$ $\boxed{\rho=0}$

15. (2) Magnetic moment $=nIA$

$$M=nI\pi R^2$$

Now $I\to I/2$ and $R\to 2R$

$$M_2=n.\frac{I}{2}\pi(2R)^2$$

$$=2nI\pi R^2=2M$$

16. (1)
17. (1) we know

$$\oint B.\overline{dl}=\mu_0 I$$

Since $$I=\frac{dq}{dt}$$

$$=\frac{d}{dt}(\sigma A)=\sigma\frac{dA}{dt}$$

$$\therefore \quad B.2\pi R=\mu_0\sigma\frac{ds}{dt}$$

$$\Rightarrow \quad B=\frac{\mu_0\sigma}{2\pi R}\frac{\overline{ds}}{dt}$$

Now surface current

$$J_s=M=\frac{B}{\mu_0}=\frac{\sigma}{2\pi R}\cdot\frac{\overline{ds}}{dt}$$

Area (s) $=\pi R^2$

$$\frac{d\bar{s}}{dt}=2\pi R\frac{dR}{dt}=2\pi R\omega R$$

$\therefore$ surface current

$$J_S=\frac{\sigma}{2\pi R}\times 2\pi R^2.\omega$$

$\boxed{J=\sigma R\omega}$

18. (3)
19. (3) force for moving charge

$$\bar{F}=q\bar{v}\times\bar{B}$$

$$BE_1=KE_2$$

$\therefore$ 1:1

20. (1) Elliptically since

$$F_x\neq F_y$$

21. (3)
22. (1)

23. (3) we know

$$\frac{E_{0r}}{E_{0i}}=\frac{n_2-n_1}{n_2+n_1}$$

$$\frac{E_{0t}}{E_{0i}}=\frac{2n_1}{n_2+n_1}$$

$$\Rightarrow \quad E_{0r}=E_{0i}\left[\frac{1-\frac{n_2}{n_1}}{\frac{n_2}{n_1}+1}\right]$$

$$=2\times\frac{1.5-1}{1.5+1}=-2\times\frac{0.5}{2.5}$$

$$=-\frac{2}{5}V/m$$

and for $E_{0t}=E_{0i}\times\dfrac{2}{1+\frac{n_2}{n_1}}$

$$=2\times2\times\frac{1}{5}\times2$$

$$\boxed{E_{0t}=\frac{8}{5}V/m}$$

24. (4)
25. (2)
26. (4)
27. (1)

28. (2) $m=2, n=1, \beta_p=12, f=6GHz$

$$\beta_p=\frac{\omega}{v}\sqrt{1-\left(\frac{f_C}{f}\right)^2}$$

$$12 = \frac{2\pi \times 6\times 10^9}{3\times 10^8}\sqrt{1-\left(\frac{f_C}{6}\right)^2}$$

$$f_c = 5.973GHz$$

29. (1) $f_r = \frac{v}{2}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2+\left(\frac{p}{c}\right)^2}$

where for TM mode to Z

$m = 1, 2, 3..., n = 1, 2, 3....., p = 0, 1, 2....$

For TE mode to Z

$m = 1, 2, 3, ...; n = 1, 2, 3...; p = 1, 2, 3...$

If $a < b < c$, then $\frac{1}{a} > \frac{1}{b} > \frac{1}{c}$

The lowest TM mode is TM_{110}, with

$$f_{r_1} = \frac{v}{2}\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{1}{b}\right)^2}$$

The lowest TE mode is TE_{011}, with

$$f_{r_2} = \frac{v}{2}\sqrt{\left(\frac{1}{b}\right)^2+\left(\frac{1}{c}\right)^2}$$

$$f_{r_1} > f_{r_2}$$

30. (2) $f_C = \frac{C}{2\sqrt{\in_r}}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2}$

$$f_{C10} = \frac{C}{2a\sqrt{\in_r}} = \frac{2\times 10^8}{2\times 1.25\times 0.06}$$

$$= 2GHz$$

31. (1) $f < f_C$

$$f < \frac{v}{2b} = \frac{3\times 10^8}{2\times b\times\sqrt{2.1}}$$

$$3\times 10^9 < \frac{3\times 10^8}{2\times b\times\sqrt{2.1}}$$

$$b < 3.4cm$$

32. $\langle v_x^2 v_y^2 v_z^2\rangle =$

$$N\left(\frac{m}{2\pi KT}\right)^{\frac{3}{2}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} v_x^2 v_y^2 v_z^2\, e^{-\frac{m\left(v_x^2 v_y^2 v_z^2\right)}{2K_B T}}\, dv_x dv_y dv_z$$

$$\frac{mv_x^2}{2KT} = x$$

$$v_x = \sqrt{\frac{2KT}{m}}.x^{1/2}$$

$$\int_{-\infty}^{\infty} v_x^2 e^{-\frac{mv_x^2}{2KT}} dv_x = \frac{2KT}{m}\int_{-\infty}^{\infty} xe^{-x}.\sqrt{\frac{2KT}{m}}\frac{1}{2}x^{-1/2}dx$$

$$= \frac{2KT}{m}\sqrt{\frac{2KT}{m}}.\int_0^{\infty} x^{\frac{1}{2}}e^{-x}\,dx$$

$$= \left(\frac{2KT}{m}\right)^{3/2}.\left\lceil\frac{3}{2}\right.$$

$$= \left(\frac{2KT}{m}\right)^{3/2}.\frac{1}{2}\sqrt{\pi}$$

$$\therefore \langle v_x^2 v_y^2 v_z^2\rangle = N\left(\frac{m}{2\pi KT}\right)^3.\left(\frac{2KT}{m}\right)^{\frac{9}{2}}.\frac{1}{8}\pi^{\frac{3}{2}}$$

$$= N\left(\frac{m}{2\pi K}\right).\left(\frac{2K}{m}\right).\frac{1}{8}\pi^{3/2}.T^3$$

$$\boxed{\langle v_x^2 v_y^2 v_z^2\rangle = \alpha T^3}$$

$$\therefore \langle v_x^2 v_y^2 v_z^2\rangle \alpha T^3$$

33. $\lambda_m T = \text{Constant}$

$$\therefore \frac{T_S}{T_P} = \frac{\lambda_P}{\lambda_S} = \frac{380}{500}$$

$$\boxed{\frac{T_S}{T_P} = 0.76}$$

34. Let heat extracted from object be dQ and temperature be T.

Efficiency of heat engine

$\eta = \left(1-\frac{T_2}{T}\right)$ (assuming maximum efficiency)

Work $dW = \eta dQ$

$$= \left(1-\frac{T_2}{T}\right)dQ$$

$$= \left(1-\frac{T_2}{T}\right)Cp\,dT$$

$\therefore$ Total work done at T_2

$$\omega = \int\left(1-\frac{T_2}{T}\right)Cp\,dT$$

$$= C_P\left[T - T_2\ln T\right]_{T_1}^{T_2}$$

$$\boxed{\omega = C_P\left(T_2 - T_1\right) - C_P T_2 \ln T_2 / T_1}$$

35. According to statistical entropy, we have

$S = K_B \ln\Omega$ ……(1)

Now number of microstates for N-identical Spin zero particles is

$\Omega_1 = (2S+1)^N = (2\times 0+1)^N = 1^N$ ……(2)

And for 2N identical spin half particles

$\therefore$ Number of microstates for N-identical spin ½ is

$$\Omega_2 = (2S+1)^N$$

$$= \left(2\times\frac{1}{2}+1\right)^N$$

$$\Omega_2 = 2^N$$

So, total number of microstates

$$\Omega = \Omega_1\Omega_2$$

$$S = K_B \ln\left(1^N.2^N\right)$$

$$= NK_B \ln 2$$

$$= 1000K_B \ln 2$$

$$= 1000K_B \times 0.6931$$

$$\boxed{S = 693.1K_B}$$

36. $\gamma = \dfrac{C_P}{C_V} = 1.4$

For adiabatic process

$$TP^{1-\gamma/\gamma} = \text{Constant}$$

$$T_i P_i^{\frac{1-\gamma}{\gamma}} = T_f P_f^{\frac{1-\gamma}{\gamma}}$$

$$T_f = \left(\frac{P_i}{P_f}\right)^{\frac{1-\gamma}{\gamma}} T_i$$

$$= T_i 2^{-2/7}$$

$$= \frac{288}{2^{2/7}} = 236.3K = -36.7^\circ C$$

37. (1)

38. $P = P_0 e^{-\alpha v}$

$$T = \frac{PV}{nR} = \frac{P_0 e^{-\alpha V}}{nR}.V$$

To maximise T, take derivative and make it equal to zero

$$\frac{dT}{dV} = \frac{P_0}{nR}\left[e^{-\alpha V} - \alpha V e^{-\alpha V}\right] = 0$$

$$\therefore V = 1/\alpha$$

Given $T = \dfrac{P_0 e^{-\alpha V}}{nR}$

For $T_{\max} = \dfrac{P_0 e^{-\alpha.\frac{1}{\alpha}}}{nR}.\dfrac{1}{\alpha}$

$$= \frac{P_0 e^{-1}}{\alpha nR}$$

$$\boxed{T_{\max} = \frac{P_0}{e\alpha nR}}$$

39. (3)

40. (1) According to Wien's displacement law

$$\lambda_m T = \text{Constant}$$

$$\Rightarrow \quad \lambda_m \propto \frac{1}{T}$$

or, $\quad v_{\max} \propto T$

So, if $v_{\max}$ becomes half, T will also becomes half.

$$T_2 = \frac{T_1}{2}$$

41. (2)

$$\frac{dQ}{dt} = \frac{KA\Delta T}{L}$$

$$\log\left(\frac{dQ}{dt}\right) = \log K + \log A + \log \Delta T - \log L$$

$$\frac{\Delta \frac{dQ}{dt}}{\frac{dQ}{dt}} = \frac{\Delta K}{K} + \frac{\Delta A}{A} + \frac{\Delta(\Delta T)}{\Delta T} + \frac{\Delta L}{L}$$

$$\frac{\Delta K}{K} = \frac{\Delta \frac{dQ}{dt}}{\frac{dQ}{dt}} - \frac{\Delta A}{A} - \frac{\Delta(\Delta T)}{\Delta T} - \frac{\Delta L}{L}$$

absolute erosion

$$\left|\frac{\Delta K}{K}\right| = \left|\frac{\Delta \frac{dQ}{dt}}{\frac{dQ}{dt}}\right| + \left|\frac{\Delta A}{A}\right| + \left|\frac{\Delta A(\Delta T)}{\Delta T}\right| - \left|\frac{\Delta L}{L}\right|$$

Assuming error in cross-section area to be zero maximum proportional error in measurement of thermal conductivity = 90%

42. (3) Let η_1 and η_2 be the efficiencies of both the heat engines

$$W_1 = \eta_1 Q_1 = \left(1 - \frac{T_2}{T_1}\right)Q_1$$

$$W_2 = \eta_2 Q_2 = \left(1 - \frac{T_3}{T_2}\right)Q_2$$

$$= \left(1 - \frac{T_3}{T_2}\right)\frac{T_2}{T_1}Q_1$$

Total work done

$$W = W_1 + W_2$$
$$= \left(1-\frac{T_2}{T_1}\right)Q_1 + \left(1-\frac{T_3}{T_2}\right)\frac{T_2}{T_1}Q$$
$$= \left[1-\frac{T_2}{T_1}+\frac{T_2}{T_1}-\frac{T_3}{T_1}\right]Q_1$$
$$= \left(1-\frac{T_3}{T_1}\right)Q_1$$

Net efficiency $\eta = \frac{W}{Q_1} = \left(1-\frac{T_3}{T_1}\right)$

43. (1) RMS velocities of the molecules are

$$V_1 = \sqrt{\frac{8kT}{\pi m}}$$
$$V_1 = \sqrt{\frac{8kT}{4\pi m}} = \frac{1}{2}\sqrt{\frac{8kT}{\pi m}}$$

Root mean square velocity of atoms in the mixture

$$V = \frac{V_1+V_2}{2} = \frac{3}{4}\sqrt{\frac{8kT}{\pi m}}$$

Since the number of molecules are equal

44. (4)

45. (4)

46. (1)

47. $H = -\mu B \sum_i S_i$

$Z = \Sigma e^{-\beta H} = \Sigma e^{+\mathrm{B}\mu B} \Sigma S_i$

$Z = e^{\mu B\beta} + e^{-\mu B\beta}$

$\langle S_i \rangle = \Sigma S_i P$

$$= \sum_i S_i \frac{e^{+\mu B\beta} \sum_i S_i}{Z}$$

$$\langle S_i \rangle = \frac{e^{\mu B\beta} - e^{-\mu B\beta}}{e^{\mu B\beta} + e^{-\mu B\beta}}$$

$$\frac{1}{3} \geq \frac{e^{\mu B\beta} - e^{-\mu B\beta}}{e^{\mu B\beta} + e^{-\mu B\beta}}$$

$e^{\mu B\beta} + e^{-\mu B\beta} \geq 3e^{\mu B\beta} - 3e^{-\mu B\beta}$

$4e^{-\mu B\beta} \geq 2e^{\mu B\beta}$

$2 \geq e^{2\mu B\beta}$

$$\ln 2 \geq 2\frac{\mu B}{K_B T} \Rightarrow \frac{1}{2}\ln 2 \geq \frac{\mu B}{K_B T}$$

48. (4)
49. (3)
50. (3)
51. (2)
52. (3)
53. (3)
54. (1)
55. (4)
56. (4)
57. (1)
58. (2)
59. (1)
60. (4)