UGC POINT

Solution (TEST SERIES 2nd PAPER) **Date:**
26 Nov 2015

1. (3)
2. (1)
Let us put charge q on inner sphere and –q on the outer sphere.
Let us take Gaussian sphere of radius r $(R_1 < r < R_2)$

$$\oint \vec{D}.\hat{n}dS = q_f$$

$$\Rightarrow D.4\pi r^2 = q$$

$$D = \frac{q}{4\pi r^2}$$

$$E = \frac{D}{\varepsilon_0\varepsilon_r} = \frac{q}{4\pi\varepsilon_0\varepsilon_r r^2}$$

Potential difference

$$V_1 - V_2 = -\int_{R_2}^{R_1} \vec{E}.\vec{dr} = -\int_{R_2}^{R_1} \frac{q}{4\pi\varepsilon_0\varepsilon_r r^2}\hat{e}_r\,\vec{dr}$$

$$= -\int_{R_2}^{R_1} \frac{q}{4\pi\varepsilon_0\varepsilon_r r^2}dr$$

$$= \left.\frac{q}{4\pi\varepsilon_0\varepsilon_r r}\right|_{R_2}^{R_1}$$

$$= \frac{q}{4\pi\varepsilon_0\varepsilon_r}\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

Capacitance $C = \frac{q}{V}$

$$= \frac{4\pi\varepsilon_0\varepsilon_r}{\frac{1}{R_1} - \frac{1}{R_2}}$$

3. (4)

The intensity of EM wave proportional to E^2 so, to reduce intensity to 1% the electric field has to be reduced to $\frac{1}{10}$ of its outside intensity

$$E(z) = \frac{E_0}{10}$$

$$\Rightarrow E_0 e^{-z/\delta} = \frac{E_0}{10}$$

$$\Rightarrow \frac{z}{\delta} = \log 10$$

$$\Rightarrow z = \delta \log_e 10$$

$$= 2.303\delta$$

Skin depth $\delta = \left(\frac{2}{\sigma\omega\mu_r\mu_0}\right)^{1/2}$

$$= \left(\frac{2}{6\times 10\times 2\pi\times 10^4\times 4\pi\times 10^{-7}}\right)^{1/2}$$

$$= \frac{1}{2\pi 10^3}\frac{1}{\sqrt{6}}$$

So, $z = 2.303\delta$
= 0.15 mm

4. (2) At any point P, let a distance x from the wire X net magnetic field

$$B = \frac{\mu_0 I}{2\pi x} - \frac{2\mu_0 I}{2\pi(d-x)} \text{ inward direction}$$

$B = 0$ if

$$\frac{\mu_0 I}{2\pi x} - \frac{2\mu_0 I}{2\pi(d-x)} = 0$$

$$\Rightarrow x = d/3$$

5. (1) The Lorentz force is perpendicular to velocity and also to the path. To move the charge in a circle of fixed radius R=2m, the external agency has to apply force perpendicular to velocity to counter increasing Lorentz, So work done is zero

6. (1) Total change Q will resides on the surface of sphere. The cavity wont effect the electric field outside the sphere.

7. (4) $Q_1 = Q_2 = Q_3 = Q$

When Q_1 is brought, there is no work done, so $\Delta_1 = 0$ when Q_2 is brought, the increase in electrostatic energy

$$\Delta_2 = \frac{Q^2}{4\pi\varepsilon_0 a}$$

When Q_3 is brought, the increases in electrostatic energy $\Delta_3 = \frac{2Q^2}{4\pi\varepsilon_0 a}$

$$\therefore \boxed{\Delta_1 : \Delta_2 : \Delta_3 = 0 : 1 : 2}$$

8. (1) $I = I_0 \cos^2\theta$

$I_1 = I_0 \left\langle \cos^2\theta \right\rangle = \frac{I_0}{2}$

$I_T = I_1 \cos^2 45 = \frac{I_0}{2} \times \frac{1}{2}$

$\boxed{I_T = \frac{I_0}{4}}$

9. Potential at centre O of cube

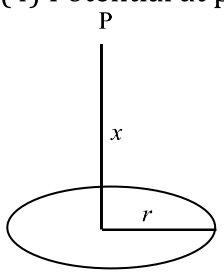
$V = \frac{\int \phi ds}{\int ds}$

$= \frac{\phi a^2}{6a^2}$

$\boxed{V = \phi / 6}$

10. (3) 11.(1) 12.(2) 13.(4)
14. (4) 15.(3) 16.(4) 17.(2)

18. (4) Potential at point P



$V = \frac{q}{4\pi\varepsilon_0 \sqrt{r^2 + x^2}}$

Ratio of potential at a height 2r on the axis to the height 3r

$= \frac{\sqrt{r^2 + 9r^2}}{\sqrt{r^2 + 4r^2}} = \sqrt{2}$

19. (4) 20.(3) 21.(3)
22. (2)
23. (2) $\lambda q + q = c(given)$

$q = \frac{c}{(\lambda + 1)}$

Figu

$F = \frac{1}{4\pi\varepsilon_0} \frac{\lambda q^2}{(\mu a - a)^2}$

$F = \frac{1}{4\pi\varepsilon_0} \frac{\lambda c^2}{(\lambda + 1)^2 (\mu a - a)^2}$

Then,

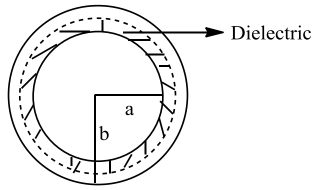
$0 = \frac{df}{d\lambda} = \frac{1}{4\pi\varepsilon_0} \frac{c^2}{(\mu a - a)^2} \left[\frac{1}{(\lambda+1)^2} - \frac{2\lambda}{(\lambda+1)^3} \right]$

Then $\lambda + 1 - 2\lambda = 0$

So $-\lambda + 1 = 0$

$\lambda = 1$

24. See figure $a < r < b$



$\varepsilon_0 E + P = 0$

$\varepsilon_0 E + P = 0$

$E = -\frac{P}{\varepsilon_0}$

$\boxed{E = -\frac{K}{r} \frac{\hat{r}}{\varepsilon_0}}$

25. (1) 26.(3) 27.(4)

28. (3) Initially identical point chares of mass m and charge q are separated by a distance a and moving at a telative speed u

The energy $E = PE + KE$

$= \frac{q^2}{4\pi\varepsilon_0 d} + \frac{1}{2}\mu u^2$ (μ = reduced mass)

$= \frac{q^2}{4\pi\varepsilon_0 d} + \frac{1}{4} m u^2 \left(\mu = \frac{m}{2} \right)$

When two charges are at large distance from each other potential energy becomes zero. Let relative velocity is v

So Energy $E = \frac{1}{2}\mu v^2 = \frac{1}{4} m v^2$

Applying law of conservation of energy

$\frac{1}{4} m v^2 = \frac{q^2}{4\pi \in_0 d} + \frac{1}{4} m u^2$

$v = \sqrt{u^2 + \frac{q^2}{\pi \in_0 md}}$

29. (4) we have

$\vec{A} = \frac{\mu_0}{4\pi} \int \frac{\vec{I}}{d} dL$

But $I = \lambda dV = \lambda d(\vec{\omega} \times \vec{r})$

$\Rightarrow \vec{A} = \frac{\mu_0 \lambda}{4\pi d} = \int (\vec{\omega} \times d\vec{r}) dL$

$\because \vec{\omega} \times d\vec{r}$ along ϕ and $|\vec{A}| \propto \frac{1}{d}$

Hence, $\vec{A}$ along $\vec{\phi}$ and $|\vec{A}| \propto \frac{1}{d}$

Now $\nabla^2 A = -\mu_0 J$

$J \to \phi$

30. (1) Equation of continuity is give as

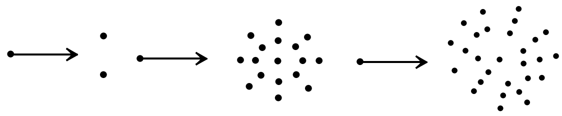
$$\frac{\partial \rho}{\partial t} + \nabla.\vec{J} = 0, \quad \frac{\partial \rho}{\partial t} + \nabla.\sigma\vec{E} = 0$$

$$\frac{\partial \rho}{\partial t} + \sigma\,\nabla.E = 0$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho}{\varepsilon} = 0 \Rightarrow \int \frac{d\rho}{\rho} + \int \frac{\sigma}{\varepsilon_0} dt = 0$$

$$\boxed{S = \rho_0(\vec{r})\exp(-\delta t/\varepsilon)}$$

31. (3) A nucleon near the surface of the nucleus, like an atom



(i) (ii) (iii)

Fig: Schematic diagram of the binding energy increment when one nucleon is added to a nucleus.

(i) The nucleon adds the binding to two neighbours.

(ii) The nucleon adds the binding to as many neighbours as it can have, the binding now has the saturation value.

(iii) A is much larger here than in (ii) but the added nucleon still has the same number of neighbours and adds only the same saturation binding energy as in (ii).

Near the surface of a liquid cannot provide the full saturation amount of binding. The number of such nucleons is proportional to the nuclear surface area $4\pi r^2$, which in turn is proportional to $A^{2/3}$. Thus a negative surface term proportional to $A^{2/3}$ appears in the formula of binding energy of the nuclei negative because the full saturation binding was already included in volume energy term $(E_V = a_V A$, where a_V is constant) and thus, the binding of the surface nucleons has been over stated. This term is not much smaller in magnitude than the volume term. The geometrical shape that corresponds to the minimum surface to volume ratio is the sphere and therefore, nuclei are nearly spherical in shape. The fit to data gives the value of the surface term as $-17.8A^{2/3} MeV$.

32. (4) Neutrons are electrically neutral. So, they cannot produce ionization in matter through which they pass. However, they can eject charged particles like the proton, ${}_1^2D$ etc. during nuclear reactions, even can also eject recoil nuclei by elastic collisions. These charged particles ejected by neutrons can produce ionization in matter through which they pass. Conventional instruments based on production of ionization can be used for the detection of these charged particles which with some modification can therefore, same as convenient tools for neutrons detection and for different types of measurements with them.

Another method of neutron detection is based on the radioactive isotope production by neutron induced nuclear reactions, especially at low neutron energies.

33. (1)

34. (1)

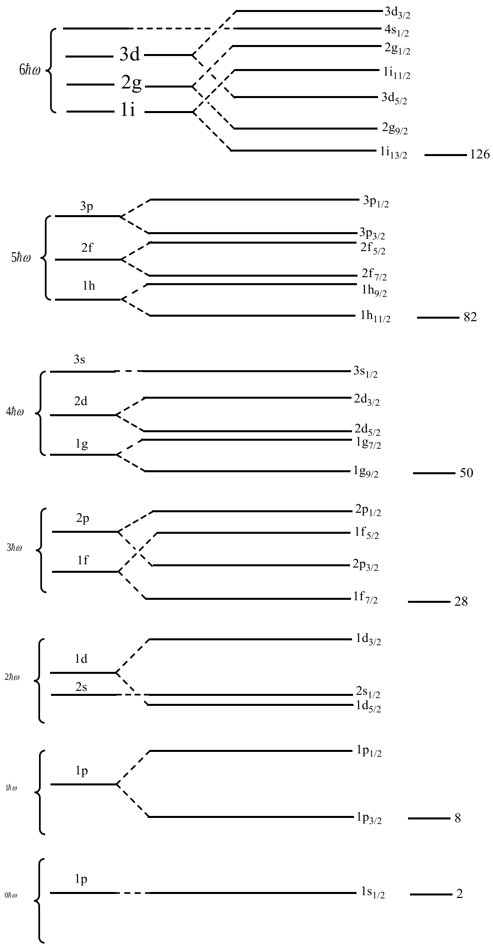


Fig: Sequence for nuclear levels according to shell model taking into account spin orbit interactions

The atomic number and atomic mass of nitrogen are 7 and 15 respectively

Hence,

p(number of protons) = 7= Z (atomic number)

= number of electrons

n(number of neutrons) = A –Z = 15 -7 = 8

According to shell model, the configuration of neutrons inside the nuclei is $1s_{1/2}1p_{3/2}1p_{1/2}$

And hence, it will not contribute anything regarding nuclear properties due to filled shell. The configuration of proton is $1s_{1/2}\,1p_{3/2}\,1p_{1/2}$ but last subshell $1p_{1/2}$ is half filled. Hence the ground state spin of ${}^{15}_{7}N$ should be $l = 1/2$. Since $l = 1$ for the P state for which the parity is odd the ground state be $\frac{1}{2}^{-}$

35. (1)
36. (3)
37. (2)

Qurak	I	I_3	S	B	Y	Q
Up-Quark u	1/2	+1/2	0	1/3	1/3	2/3
Down-Quark d	1/2	−1/2	0	1/3	1/3	−1/3
Strange-Quark s	0	0	−1	1/3	−2/3	−1/3
Charm-Quark G	0	0	0	1/3	1/3	2/3
Bottom-Quark b	0	0	0	1/3	1/3	−1/3
Top-Quark t	(0)	(0)	(0)	(+1/3)	(+1/3)	−2/3

It is clear from the above table that the quark model of the proton is uud.

38. (3)

39. (1) Experimental evidence show that the distribution of nuclear matter is nearly uniform, so that the nuclear matter density ρ_m is also approximately constant. Since, nuclear mass is almost linear proportional to the mass number A, this means that

$\rho_m \sim A/V$ = constant

i.e., the nuclear volume $V \propto A$. Assuming a spherical shape of the nucleus with a radius R,

We get

$$V = \frac{4}{3}\pi R^3 \propto A$$

Or $$R \propto A^{1/3}$$

$$R = r_0 A^{1/3} \quad \text{............ (i)}$$

Where, r_0 is a constant known as the nuclear radius parameter?

For ${}^{16}_{8}O$, the volume $V = \frac{4}{3}\pi R_0^3$ ....(ii)

From equation (i) and (ii) we have

$$V_0 = \frac{4}{3}\pi\left(r_0^3 A\right)$$

And charge radius of oxygen is

$$R_0 = r_0(128)^{1/3} \quad \text{............. (iii)}$$

Similarly, for ${}^{128}_{54}Xe$, the charge radius R_{Xe} is

$$R_{Xe} = r_0(128)^{1/3} \quad \text{............. (iv)}$$

From (iv) and (iii), we have

$$\frac{R_{Xe}}{R_0} = \left(\frac{128}{16}\right)^{1/3} = 2 \Rightarrow R_{Xe} = 2R_0$$

Therefore, the volume of ${}_{54}Xe^{128}$ is

$$V_{Xe} = \frac{4}{3}\pi(2R_0)^3$$

$$= 8\left(\frac{4}{3}\pi R_0^3\right) = 8V$$

40. (3)

41. (4)With the solution given in Q. 34

For ${}^{17}_{8}O$

The number of protons = 8 = Z

The number of neutrons = 9 = N

The proton configuration is $1s_{1/2}, 1p_{3/2}1p_{1/2}$,hence the al subshells are completely filled. The neutron configuration is $1s_{1/2}, 1p_{3/2}, 1p_{1/2}$ and last unpaired neutron will go in $1d_{5/2}$ sub shell which is partially filled. Hence the ground state spin of $^{17}_{8}O$ should be $l = 5/2$. Since, $l = 2$ for the d state for which the parity is even. Therefore, the ground state should be $\frac{5}{2}^+$

42. (1)
43. (2)
44. (3) The product of the uncertainty Δx in the position of a body at some instant and uncertainty Δp in its momentum at the same instant is equal to or greater than $\hbar$ i.e.,
$\Delta x \Delta p > \hbar$. The uncertainty relation $\Delta x\, \Delta p \geq \hbar$ can also be written in terms of the conjugate pair of quantities energy E and time t. we know that the kinetic energy of a particle is

$$E = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

Where, p is the linear momentum $(p = mv)$. Thus,

$$\Delta E = \frac{p\Delta p}{m}$$

But $\quad p = mv = m\frac{\Delta x}{\Delta t}$

$\therefore \quad \Delta E = \frac{\Delta x}{\Delta t}\Delta p$

Or $\quad \Delta E \Delta t = \Delta x \Delta p$

Since, $\quad \Delta x \Delta p \geq \hbar$

We can write

$$\Delta E \Delta t \geq h/2\pi$$

In question, $\Delta t = 10^{-9} s$

Hence, $\quad \Delta E = \frac{6.6\times10^{-34}}{2\times3.14\times10^{-19}} J$

$$= \frac{6.6\times10^{-34}}{6.28\times10^{-9}\times1.6\times10^{-19}}$$

$$\approx 0.65\times10^{-6} eV \approx 10^{-6} eV$$

45. (1)
46. (3)

Particles	Strangeness number (S)	Isospin number (I)	Isospin Component (I_3)
Proton (p)	0	1/2	1/2
Neutron (n)	0	1/2	-1/2
Lambda (Λ^0)	0	0	0
(Sigma) Σ^+	-1	1	+1
Σ^-	-1	1	-1
Σ^0	-1	1	0
$(X_i)\Xi^0$	-2	1/2	1/2
Ξ^-	-2	1/2	-1/2
Omega Ω^-	-3	0	0
$P_{ionx}\pi^+$	0	1	+1
π^-	0	1	-1
π^0	0	1	0
$K_{ion}K^+$	+1	1/2	1/2
K^0	+1	1/2	-1/2
$Eta\ \eta^0$	0	0	0

The given nuclear reaction is

$$\pi^- + \rho \rightarrow K^- + \Sigma^+$$

For l_3 -1 ½ -1/2 1

$\Delta l_3 = 0$

For S, 0 0 -1 -1

$\Delta S \neq 0$

Baryon number B 0 1 0 1

$\Delta B = 0$

From above, it is clear that the process is not allowed because $\Delta S \neq 0$

47. (3)
48. (1)From the solution given in Q.34 the neutron configuration of $^{9}_{4}$Be (total number of neutrons in $^{9}_{4}$Be is 5) is $(1s_{1/2})^2(1p_{3/2})^3$. The last neutron will be unpaired and go in $1p_{3/2}$ sub shell. Hence the ground state spin of $^{9}_{4}Be$ nucleus is $J = 3/2$

49. (1) $E = mc^2$ for $^{11}_{6}C$

$$E_1 = [6m_p + 5m_n - 11]c^2$$

$$E_2 = [5m_p + 6m_n - 11]c^2 \text{ for } ^{11}_{5}B$$

$$E_1 - E_2 = (m_p - m_n)c^2 = \Delta MeV$$

Now,

$$E_O = [8m_p + 9m_n - 17]c^2$$

$$E_F = (9m_p + 8m_n - 17)c^2$$

$$E_F - E_O = [m_p - m_n]c^2$$

$\Rightarrow \quad E_F = E_0 + (m_p - m_n)c^2$

$$= E_0 + \Delta MeV$$

$\Rightarrow \quad E_F - E_O = [938.27 - 939.57]$

$$E_F - E_O = -1.30$$

$$E_F = E_O - 1.30 \approx 1.39 MeV$$

50. (4)