UGC POINT

SOLUTION (TEST SERIES 3RD PAPER) DATE: 29 NOV 2015

1. First excited state

$$\Psi_2(x_1x_2)=\frac{1}{\sqrt{2}}\left[\Psi_1(x_1).\Psi_2(x_2)+\Psi_2(x_1)\Psi_1(x_2)\right]$$

$$\Psi_2(x,t)=\frac{1}{\sqrt{2}}\frac{2}{a}\left[\sin\frac{\pi x_1}{a}.\sin\frac{2\pi x_2}{a}+\sin\frac{2\pi x_1}{a}\sin\left(\frac{\pi x_2}{a}\right)\right]$$

$$E_2^{(1)}=-aV_0\left\langle\Psi_2\left|\delta(x_1-x_2)\right|\Psi_2\right\rangle$$

$$-aV_0\frac{1}{2}\left(\frac{2}{a_0}\right)^2\int_0^a\int_0^a\left[\sin\left(\frac{\pi x_1}{a}\right)\sin\left(\frac{2\pi x_2}{a}\right)+\sin\left(\frac{2\pi x_1}{a}\right)\sin\left(\frac{\pi x_2}{a}\right)\right]^2\delta(x_1-x_2)dx_1dx_2$$

$$=-\frac{2V_0}{a}\int_0^a\left[\sin\left(\frac{\pi x}{a}\right)\sin\left(\frac{2\pi x}{a}\right)+\sin\left(\frac{2\pi x}{a}\right)\sin\left(\frac{\pi x}{a}\right)\right]^2dx$$

$$=-\frac{2V_0}{a}\times4\int_0^a\sin^2\frac{\pi x}{a}.\sin^2\left(\frac{2\pi x}{a}\right)dx$$

Let $\frac{\pi x}{a}=y\Rightarrow dx=\frac{a}{\pi}dy$

$$=-\frac{8V_0}{a}\frac{a}{\pi}\int_\pi^a\sin^2 y\sin^2 2y dy$$

$$=-\frac{8V_0}{a}\frac{a}{\pi}\int_0^\pi\sin^2 y(2\sin y\cos y)^2dy$$

$$=-\frac{32V_0}{\pi}\int_0^\pi\sin^4 y\cos^2 y\,dy$$

$$=-\frac{32V_0}{\pi}\int_0^\pi\sin^4 y(1-\sin^2 y)dy$$

$$=-\frac{32V_0}{\pi}\left[\int_0^\pi(\sin^4 y-\sin^6 y)dy\right];\qquad\left[\int_0^\pi\sin^m\theta.\sin^n\theta d\theta=\frac{\left\lceil\frac{m+1}{2}.\right\lceil\frac{n+1}{2}}{2.\left\lceil\frac{m+n+2}{2}\right.}\right]$$

$$-\frac{32V_0}{\pi}\left(\frac{3\pi}{8}-\frac{5\pi}{16}\right)$$

$$\boxed{E_2^1=-2V_0}$$

2. Bohr quantization formula is

$$\oint P(x)\,dx=n\pi\hbar$$

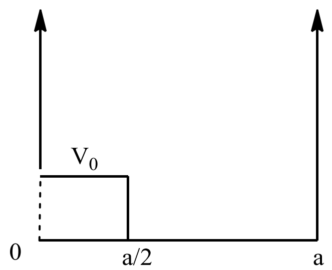
$$P(x)=\sqrt{2m(E-V(x))}$$

$$\int_0^{a/2}\sqrt{2m(E-V_0)}dx+\int_{a/2}^a\sqrt{2mE}dx=n\pi\hbar$$

$$\sqrt{2m(E-V_0)}(a/2)+\sqrt{2mE}(a/2)=n\pi\hbar$$

$$\sqrt{2m}(a/2)\left(\sqrt{E}+\sqrt{E-V_0}\right)=n\pi\hbar$$

$$\sqrt{E}+\sqrt{E-V_0}=2n\pi\hbar/\sqrt{2m}a$$



On squaring $E+E-V_0+2\sqrt{E\left(E-V_0\right)}=\frac{4n^2\pi^2\hbar^2}{2ma^2}$

$$2\sqrt{E\left(E-V_0\right)}=4E_n^0-2E+V_0$$

On squaring both side

$$4E\left(E-V_0\right)=16E_n^{02}+4E^2+V_0^2-16E_n^0E-4EV_0+8E_n^0V_0$$

$$4E^2-4EV_0=16E_0^{02}+4E+V_0^2-16E_n^0E-4EV_0+8E_n^0V_0$$

$$16E_n^0E=16E_n^{02}+8E_n^0V_0+V_0^2$$

$$E=E_n^0+\frac{V_0}{2}+\frac{V_0^2}{16E_n^0}$$

3. $\psi\left(x\right)=Ae^{-bx^2}$

$$V\left(x\right)=\frac{1}{2}m\omega^2x^2$$

$$|A|^2\int_{-\infty}^{\infty}e^{-2bx^2}dx=1$$

$$2.|A|^2.\frac{1}{2}\frac{\left\lceil\frac{0+1}{2}\right.}{\left(2b\right)^{\frac{0+1}{2}}}=1 \quad ; \quad \int_0^{\infty}x^me^{-\alpha x^n}dx=\frac{1}{n}\frac{\left\lceil\frac{m+1}{2}\right.}{\left(\alpha\right)^{\frac{m+1}{n}}}$$

$$|A|^2.\frac{\lceil 1/2}{\left(2b\right)^{1/2}}=1$$

$$|A|^2=\frac{\sqrt{\pi}}{\left(2b\right)^{1/2}}=1$$

$$\boxed{A^2=\sqrt{\frac{2b}{\pi}}} \quad \Rightarrow A=\left(\frac{2b}{\pi}\right)^{1/4}$$

$$\langle T\rangle=\left\langle\frac{p^2}{2m}\right\rangle=-\frac{\hbar^2}{2m}\int_{-\infty}^{\infty}A^2e^{-bx^2}\frac{\partial^2}{\partial x^2}e^{-bx^2}dx$$

$$=-\frac{\hbar^2}{2m}\times 2b^2\int_0^{\infty}A^2e^{-bx^2}.e^{-bx^2}dx$$

$$=-\frac{\hbar^2b^2}{m}A^2.\int_0^{\infty}e^{-2bx^2}dx \qquad \boxed{\langle T\rangle=\frac{\hbar^2b}{2m}}$$

$$\langle V\rangle=2\alpha A^2\int_0^{\infty}x^4e^{-2bx^2}$$

$$=2\alpha A^2\frac{3}{8\left(2b\right)^2}\sqrt{\frac{\pi}{2b}}$$

$$=\frac{3\alpha}{16b^2}\sqrt{\frac{\pi}{2b}}.\sqrt{\frac{2b}{\pi}}=\frac{3\alpha}{16b^2}$$

$$\langle H\rangle=\frac{\hbar^2b}{2m}+\frac{3\alpha}{16b^2}$$

$$\frac{\partial H}{\partial b}=\frac{\hbar^2}{2m}-\frac{3\alpha}{8b^3}=0$$

$$b^3=\frac{3\alpha m}{4\hbar^2}$$

$$b=\left(\frac{3\alpha m}{4\hbar^2}\right)^{1/3}$$

$$\langle H\rangle=\frac{\hbar^2}{2m}\left(\frac{3\alpha m}{4\hbar^2}\right)^{1/3}+\frac{3\alpha}{16}\left(\frac{4\hbar^2}{3\alpha m}\right)^{2/3}$$

$$=\frac{\alpha^{1/3}\hbar^{4/3}}{m^{2/3}}3^{1/3}4^{-1/3}\left(\frac{1}{2}+\frac{1}{4}\right) \quad \boxed{H_{\min}=\frac{3}{4}\left(\frac{3\alpha\hbar^4}{4m^2}\right)^{1/3}}$$

4. $X_1=\begin{pmatrix}1\\0\\0\end{pmatrix}$, Eigen value V_0, $X_2=\begin{pmatrix}0\\1\\0\end{pmatrix}$, Eigen value V_0, $X_3=\begin{pmatrix}0\\0\\2\end{pmatrix}$, Eigen value = $2V_0$

Here exist 2-fold degeneracy

$$\begin{vmatrix}H'_{11}-E' & H'_{12}\\ H'_{21} & H'_{22}-E'\end{vmatrix}=0$$

$$H_{11}=\langle X_1|H'|X_1\rangle=\in V_0(1\,0\,0)\begin{pmatrix}-1&0&0\\0&0&1\\0&1&0\end{pmatrix}\begin{pmatrix}1\\0\\0\end{pmatrix}=\in V_0(1\,0\,0)\begin{pmatrix}-1\\0\\0\end{pmatrix}$$

$$\boxed{H_{11}=-\in V_0}$$

$$H_{22}=\langle X_2|H'|X_2\rangle=\in V_0(0\,1\,0)\begin{pmatrix}-1&0&0\\0&0&1\\0&1&0\end{pmatrix}\begin{pmatrix}0\\1\\0\end{pmatrix}$$

$$H_{22}=\in V_0(0\,1\,0)\begin{pmatrix}0\\0\\1\end{pmatrix}=0$$

$$H_{12}=\langle X_1|H'|X_2\rangle=\in V_0(1\,0\,0)\begin{pmatrix}-1&0&0\\0&0&1\\0&1&0\end{pmatrix}\begin{pmatrix}0\\1\\0\end{pmatrix}=\in V_0(1\,0\,0)\begin{pmatrix}0\\0\\1\end{pmatrix}=0$$

$$\begin{vmatrix}-\in V_0-E' & 0\\ 0 & 0-E'\end{vmatrix}=0$$

$$-(\in V_0+E')(-E')=0$$

$$(\in V_0+E')(E')=0$$

$$E'=0$$

$$E=-\in V_0$$

$$\boxed{E_1=V_0-\in V_0} \quad \boxed{E_2=V_0}$$

5. For first excited state $\left.\begin{matrix}|1\rangle-E_{001}\\|2\rangle-E_{010}\\|3\rangle-E_{100}\end{matrix}\right\}\to\frac{5}{2}\hbar\omega$ 3-fold degeneracy

Matrix element will be 3×3

$$\begin{vmatrix}H'_{11}-\in' & H'_{12} & H'_{13}\\ H_{21} & H_{22}'-\mathrm{E}' & H'_{22}\\ H'_{31} & H'_{32} & H'_{33}-E'\end{vmatrix}=0$$

$H'_{11} = \lambda \left\langle 001 \middle| x^2 yz \middle| 001 \right\rangle = 0$

$H'_{22} = H'_{33} = 0$

$H_{13} = H_{31} = H_{32} = H_{23} = 0$

$H_{21} = \left\langle 010 \middle| x^2 yz \middle| 001 \right\rangle$

$= \left(\dfrac{\hbar}{2m\omega} \right)^2$

6. The differential scattering crossection in Born approximation is given by

$\dfrac{d\sigma}{d\Omega} = \left| f(\theta,\phi) \right|^2$; where $f(\theta,\phi)$ is the scattering amplitude

Now $f(\theta,\phi) = -\dfrac{2m}{\hbar^2 K} \int_0^\infty r \sin Kr V(r) dr$ where $K = 2K \sin\theta/2$

Given that $V(r) = \begin{cases} V_0; & r \le R \\ 0\ ; & r < R \end{cases}$

$$\left| f(\theta,\phi) \right|^2_{V=V_0} = \left(\frac{2mV_0}{\hbar^2 K} \right)^2 \left| \int_0^R r \sin Kr dr \right|^2 \qquad \ldots\ldots\ldots\ldots (1)$$

When potential V_0 is increased by $2V_0$, then

$$\left| f(\theta,\phi) \right|^2_{V=2V_0} = \frac{(2m2V_0)^2}{(\hbar^2 K)^2} \left| \int_0^R r \sin Kr dr \right|^2 \qquad \ldots\ldots\ldots\ldots (2)$$

Now divide equation (2) by (1) we get

$$\frac{\left| f(\theta,\phi) \right|^2_{V=2V_0}}{\left| f(\theta,\phi) \right|^2_{V=V_0}} = 4$$

$$\frac{\left(\frac{d\sigma}{d\Omega} \right)_{V=2V_0}}{\left(\frac{d\sigma}{d\Omega} \right)_{V=V_0}} = 4 \qquad \boxed{\left(\frac{d\sigma}{d\Omega} \right)_{V=2V_0} = 4 \left(\frac{d\sigma}{d\Omega} \right)_{V=V_0}}$$

7. Applying uncertainty principle

$\Delta P \Delta Y = \hbar$

$\Delta(mV_y)\Delta y = \hbar$

$\Delta V_y = \dfrac{\hbar}{m\Delta y}$

$\Delta V_y = \dfrac{6.6 \times 10^{-34}}{9.1 \times 10^{-31} \times 2 \times 1 \times 10^{9}}$

$= 1.166 \times 10^5 m/\sec$

8. (3) degeneracy in 2D Harmonic oscillator is $g_n = (n+1)$

due to electrons $g_n = (n+1) \times 2$

$g_0 = 2$

$g_1 = 4$

$E_n = (n+1)\hbar\omega$

$E = 2 \times \hbar\omega + 4 \times 2\hbar\omega$

$E = 10\hbar\omega$

9. (3)

10. $\left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}+V\right)\left(\alpha x e^{-\beta x}e^{i\gamma t/\hbar}\right)=-\frac{\hbar\partial}{\partial t}\left(\alpha x e^{-\beta x}e^{i\gamma t/\hbar}\right)$ ....(1)

$$\frac{d}{dx}\left(\alpha x e^{-\beta x}\right)=\alpha e^{-\beta x}-\alpha\beta x e^{-\beta x}$$

$$\frac{d^2}{dx}\left(\alpha x e^{-\beta x}\right)=-\alpha\beta e^{-\beta x}-\alpha\beta e^{-\beta x}+\alpha\beta^2 x e^{-\beta x}$$

$$=-2\alpha\beta e^{-\beta x}+\alpha\beta^2 x e^{-\beta x}$$

Now from (1)

$$\left[-\frac{\hbar^2}{2m}\left(-2\beta+\beta^2 x\right)+V\right]\alpha e^{-\beta x}e^{+\gamma t/\hbar}=i\hbar\frac{(i\gamma)}{\hbar}\alpha x e^{-\beta x}e^{i\gamma t/\hbar}$$

$$\Rightarrow\frac{\hbar^2}{2m}\left(2\beta-\beta^2 x\right)+V=-\gamma x$$

$$V=-\frac{\hbar^2\beta}{m}+\left(\frac{\beta^2\hbar^2}{2m}-\gamma\right)x$$

$\boxed{V=K_1+K_2 x}$ Where $K_1=-\frac{\hbar^2\beta}{m},\quad K_2=\left(\frac{\beta^2\hbar^2}{2m}-\gamma\right)$

11. $\Psi(r,t)=\frac{4}{5}\phi_1 e^{-iE_1 t/\hbar}+\frac{3}{5}\phi_2 e^{-iE_2 t/\hbar}$

$$=e^{-iE_1 t/\hbar}\left[\frac{4}{5}\phi_1+\frac{3}{5}\phi_2 e^{i(E_1-E_2)t/\hbar}\right]$$

$$=e^{-iE_1 t/\hbar}\left[\frac{4}{5}\phi_1+\frac{3}{5}\phi_2 e^{i\pi}\right]$$

$$=e^{-iE_1 t/\hbar}\left[\frac{4}{5}\phi_1-\frac{3}{5}\phi_2\right]$$

So wave function at time $t=\frac{h}{2(E_1-E_2)}$ (accurate to within a phase)

$$\Psi(x,t)=\frac{4}{5}\phi_1-\frac{3}{5}\phi_2$$

12. $A+\gamma=B$

$$\gamma=B-A$$

$$hv=(M+\Delta)C^2-MC^2$$

$$v=\frac{\Delta C^2}{h}$$

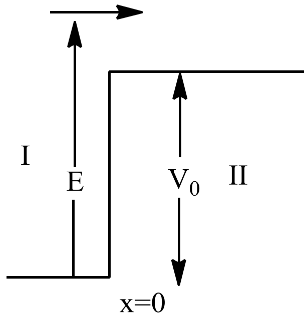
13. In region I

$\Psi_1=Ae^{ikx}+Be^{-ikx}$ g

In region II

$\Psi_2=ce^{ikx}$

$K=\sqrt{\frac{2mE}{\hbar^2}},\ K'=\sqrt{\frac{2m(E-V_0)}{\hbar^2}}$



$\Psi_1|_{x=0}=\Psi_2|_{x=0}$

A +B =C ....(1)

$$\left.\frac{\partial\Psi_1}{\partial x}\right|_{x=0}=\left.\frac{\partial\Psi_2}{\partial x}\right|_{x=0}$$

$$ik(A-B)=ik'c$$

$A-B=\frac{K'}{K}c$ .... (2)

Solving (1) and (2)

$$A=\frac{1}{2}\left(1+K'/K\right)c$$

$$B=\frac{1}{2}\left(1-K'/K\right)c$$

$$\frac{B}{A}=\frac{K-K'}{K+K'}$$

$$\frac{B}{A}=\frac{1-\sqrt{1-V_0/E}}{1+\sqrt{1+V_0/E}}=0.4$$

Reflection coefficient

$$R=\left|\frac{B}{A}\right|^2=0.16$$

Transmission coefficient

$$T=1-R=0.84$$

14. According to the question

$$H=\frac{P_1^2}{2m}+\frac{P_2^2}{2m}+\frac{1}{2}m\omega^2\left(r_1^2+r_2^2\right)+K\sigma_1.\sigma_2$$

Given $\hbar\omega=0.1eV$ and $K=0.2eV$

We known that

$$mvr=\hbar$$

$$mv=\frac{\hbar}{r}$$

$$mv^2=\hbar\omega$$

$$(mv)^2=m\hbar\omega\Rightarrow P^2=m\hbar\omega$$

$$H=\frac{m\hbar\omega}{2m}+\frac{m\hbar\omega}{2m}+\frac{2\times m\hbar\omega}{2m}+2\sigma_1.\sigma_2$$

$$=4.\frac{m\hbar\omega}{2m}+K\sigma_1\sigma_2$$

$$\sigma_1=\begin{bmatrix}0&1\\1&0\end{bmatrix},\quad \sigma_2=\begin{bmatrix}0&-i\\i&0\end{bmatrix}$$

$$\sigma_1\sigma_2=\begin{bmatrix}0&1\\1&0\end{bmatrix}\begin{bmatrix}0&-i\\i&0\end{bmatrix}=-5/2$$

Thus $H=2\left(0.1eV\right)+K\sigma_1\sigma_2$

$$=2\left(0.1eV\right)+\left(0.2eV\right)\left(-5/2\right)$$

$$H=-0.3eV$$

15. $\hat{A}=\lambda\sigma.B$

$$=\lambda\left[\sigma_xB_x+\sigma_yB_y+\sigma_zB_z\right]$$

$$=\lambda\left[\begin{bmatrix}0&1\\1&0\end{bmatrix}\frac{B}{\sqrt{2}}+\begin{bmatrix}0&-i\\i&0\end{bmatrix}\frac{B}{\sqrt{2}}\right]$$

$$=\frac{\lambda B}{\sqrt{2}}\begin{bmatrix}0&1-i\\1+i&0\end{bmatrix}$$

$$=\begin{bmatrix}0&(1-i)\lambda B/\sqrt{2}\\(1+i)\lambda B/\sqrt{2}&0\end{bmatrix}$$

For Eigen value

$$\begin{bmatrix} -K & (1-i)\lambda B/\sqrt{2} \\ (1+i)\lambda B/\sqrt{2} & -K \end{bmatrix} = 0$$

$$K^2 - \left(\lambda B/\sqrt{2}\right)^2 \left(1-i^2\right) = 0$$

$$K^2 = (\lambda B)^2 / 2 \times 2$$

$$k^2 = (\lambda B)^2$$

$$\boxed{K = \pm \lambda B}$$

16. $\left[\hat{A},\left[\hat{B},\hat{C}\right]\hat{D}\right] = \left[\hat{B},\hat{C}\right]\left[\hat{A},\hat{D}\right] + \left[\hat{A},\left[\hat{B},\hat{C}\right]\right]\hat{D}$

$$= \left(\hat{B}\hat{C} - \hat{C}\hat{B}\right)\left(\hat{A}\hat{D} - \hat{D}\hat{A}\right) + \hat{A}\left(\hat{B}\hat{C} - \hat{C}\hat{B}\right)\hat{D} - \left(\hat{B}\hat{C} - \hat{C}\hat{B}\right)\hat{A}\hat{D}$$

$$= \hat{C}\hat{B}\hat{D}\hat{A} - \hat{B}\hat{C}\hat{D}\hat{A} + \hat{A}\hat{B}\hat{C}\hat{D} - \hat{A}\hat{C}\hat{B}\hat{D}$$

17. $\left\langle \hat{B} \right\rangle = \dfrac{\left\langle \Psi \middle| \hat{B} \middle| \Psi \right\rangle}{\left\langle \Psi | \Psi \right\rangle}$

$$\left\langle \Psi | \Psi \right\rangle = \left(\frac{1}{\sqrt{2}} \langle \phi_1 | + \frac{1}{\sqrt{5}} \langle \phi_2 | + \frac{1}{\sqrt{10}} \langle \phi_2 | \right) \left(\frac{1}{\sqrt{2}} | \phi_1 \rangle + \frac{1}{\sqrt{5}} | \phi_2 \rangle + \frac{1}{\sqrt{10}} | \phi_3 \rangle \right)$$

$$= \frac{8}{10}$$

$$\left\langle \Psi \middle| \hat{B} \middle| \Psi \right\rangle = \left(\frac{1}{\sqrt{2}} \langle \phi_1 | + \frac{1}{\sqrt{5}} \langle \phi_2 | + \frac{1}{\sqrt{10}} \langle \phi_3 | \right) B \left(\frac{1}{\sqrt{2}} | \phi_1 \rangle + \frac{1}{\sqrt{5}} | \phi_2 \rangle + \frac{1}{\sqrt{10}} | \phi_3 \rangle \right)$$

$$= \frac{1}{2} + \frac{2^2}{5} + \frac{3^2}{10}$$

$$= 22/10$$

$$\left\langle \hat{B} \right\rangle = \frac{22/10}{8/10} = 22/8 \qquad \boxed{\left\langle \hat{B} \right\rangle = 11/4}$$

18. $(-1)^3 \dfrac{d^3}{d\theta^3} \cos(3\theta)\Big|_{\theta=\pi/2}$

$$(-1)^3\, 27 \sin(3\pi/2)$$

$$= 27$$

19. $\hat{P}_H(t) = e^{it\hat{H}/\hbar} \hat{P} e^{-it\hat{H}/\hbar}$

$$= \hat{P} + \frac{it}{\hbar}\left[\hat{H},\hat{P}\right] + \frac{1}{2!}\left(\frac{it}{\hbar}\right)^2 \left[\hat{H},\left[\hat{H},\hat{P}\right]\right] + \ldots\ldots$$

$$= \hat{\mathrm{P}}\left[1 - \frac{(\omega t)^2}{2!} + \frac{(\omega t)^4}{4!} + \ldots\right] - m\omega\hat{x}\left[(\omega t) - \frac{(\omega t)^3}{3!} + \frac{(\omega t)^5}{5!} + \ldots\right]$$

$$\boxed{\hat{P}_H(t) = \hat{P}\cos(\omega t) - m\omega\hat{x}\sin(\omega t)}$$

20. With the perturbation potential given we get

$$C(1S \to 2P) = \frac{eE_0}{i\hbar} \left\langle \phi_{210} | 2 | \phi_{100} \right\rangle \int_0^\infty e^{i\omega t} e^{-\gamma t}\, dt$$

Where $\omega = (E_{21} - E_{10})$, the integral yields $\dfrac{1}{\gamma - i\omega}$ so that the absolute sequence of $C(1S \to 2P)$ is

$$P(1S \to 2P) = \frac{e^2 E_0^2 \left|\langle \phi_{210} | z | \phi_{100} \rangle\right|^2}{\hbar^2 \left(\omega^2 + \gamma^2\right)}$$

Using $\left|\langle \phi_{210} | z | \phi_{100} \rangle\right|^2 = \frac{2^{15}}{3^{10}} a_0^2$

$$\therefore P(1S \to 2P) = \frac{2^{15}}{3^{10}} \frac{e^2 E_0^2 a_0^2}{h^2 \left(\omega^2 + \gamma^2\right)}$$

21 (4)

22. (3) Reflection coefficient is given by

$$R = \left|\frac{B}{A}\right|^2 = \left(\frac{\sqrt{E} - \sqrt{E - V_0}}{\sqrt{E} + \sqrt{E + V_0}}\right)^2 = \frac{1}{16}$$

$$\left|\frac{B}{A}\right| = \left(\frac{5-3}{5+3}\right) = \frac{2}{8}$$

$$|B| = \frac{1}{4}|A|$$

$$T = 1 - \frac{1}{10} = \frac{15}{16}$$

$$T = \left|\frac{C}{A}\right|^2 \frac{k_2}{k_1} = \left|\frac{C}{A}\right|^2 \frac{\sqrt{E - V_0}}{\sqrt{E}}$$

$$\left|\frac{C}{A}\right|^2 = \frac{15}{16} \times \frac{5}{3}$$

$$\left|\frac{C}{A}\right| = \left(\frac{75}{48}\right)^{1/2}$$

$$|C| = 1.24|A|$$

23.(1) $\pi|r\rangle = |-r\rangle$

$$\pi.\pi|r\rangle = \pi|-r\rangle$$

$$= |r\rangle$$

$$\pi^2 = 1$$

$$P_+ P_- = 1/4\left[1 + \pi - \pi - \pi^2\right]$$

$$P_+ P_- = 0$$

$$P_- P_+ = 0$$

$$\pi P_+ = \frac{\pi}{2}[1 + \pi]$$

$$= \frac{1}{2}(\pi + \pi^2)$$

$$\pi P_+ = \frac{1}{2}(1 + \pi) = P_+$$

$$\pi P_- = \pi \frac{1}{2}(1 - \pi)$$

$$= \frac{1}{2}(\pi - \pi^2)$$

$$= \frac{1}{2}(\pi - 1)$$

$$= -\frac{1}{2}(1 - \pi)$$

$$= -P_-$$

24. (4) $\left[K,V\right]=\left[\frac{p_x^2}{2m},\frac{1}{2}m\omega^2x^2\right]$

$$=\frac{1}{4}\omega^2\left[p_x^2,x^2\right]$$

$$=\frac{1}{4}\omega^2\left[p_x\left(p_x,x^2\right)+\left(P_x,x^2\right)P_x\right]$$

$$=\frac{\omega^2}{4}\left[\left(-2i\hbar\right)\left(p_xx+xp_x\right)\right]$$

$$=-\frac{i\omega^2}{2}\hbar\left[p_xx+xp_x\right]$$

25. (3) $\langle\Psi|A|\Psi\rangle$

$$\frac{d}{dt}\langle A\rangle=\frac{d}{dt}\langle\Psi|A|\Psi\rangle$$

$$\frac{d}{dt}\langle\Psi|A|\Psi\rangle+\left\langle\Psi\left|\frac{\partial A}{\partial t}\right|\Psi\right\rangle+\left\langle\Psi\left|A\frac{d}{dt}\right|\Psi\right\rangle$$

$$H\left|\Psi\right\rangle=i\hbar\frac{d}{dt}\left|\Psi\right\rangle$$

$$\left\langle\Psi\right|H=-i\hbar\frac{d}{dt}\left\langle\Psi\right| \qquad \left\langle\Psi\right|=i\hbar\frac{d}{dt}\left\langle\Psi\right|$$

$$-\frac{1}{i\hbar}\langle\Psi|\mathrm{H}A|\Psi\rangle+\left\langle\Psi\left|\frac{\partial A}{\partial t}\right|\Psi\right\rangle+\frac{1}{i\hbar}\left\langle\Psi\right|AH\left|\Psi\right\rangle$$

$$=\frac{d\langle A\rangle}{dt}$$

$$\frac{d\langle A\rangle}{dt}=\frac{1}{i\hbar}\left[A,H\right]+\left\langle\frac{\partial A}{\partial t}\right\rangle$$

$$\left[A,H\right]=\frac{1}{i\hbar}\left[\frac{d}{dt}\langle A\rangle-\left\langle\frac{\partial A}{\partial t}\right\rangle\right]$$

26. $\sigma_x=\begin{pmatrix}0&1\\1&0\end{pmatrix},\ \sigma_y=\begin{pmatrix}0&-i\\i&0\end{pmatrix},\sigma_z=\begin{pmatrix}1&0\\0&-1\end{pmatrix}$

$$\sigma_x\sigma_y\sigma_z=\begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}0&-i\\i&0\end{pmatrix}\begin{pmatrix}1&0\\0&-1\end{pmatrix}$$

$$=\begin{pmatrix}i&0\\0&-i\end{pmatrix}\begin{pmatrix}1&0\\0&-1\end{pmatrix}$$

$$=\begin{pmatrix}i&0\\0&i\end{pmatrix}$$

Eigen value i,i and Eigen vector $\begin{pmatrix}1\\0\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}$

27. $\langle\chi|S_x|\chi\rangle$

$$\left\langle\chi\left|\frac{S_++S_-}{2}\right|\chi\right\rangle$$

$$=\frac{1}{8}\left\langle\chi_{1/2}+\sqrt{3}\chi_{-1/2}\right|S_++S_-\left|\chi_{1/2}+\sqrt{3}\chi_{-1/2}\right\rangle$$

$$=\frac{1}{8}\left[\left\langle\chi_{1/2}+\sqrt{3}\chi_{-1/2}\right|S_+\left|\chi_{1/2}+\sqrt{3}\chi_{-1/2}\right\rangle+\left\langle\chi_{1/2}+\sqrt{3}\chi_{-1/2}\right|S_-\left|\chi_{1/2}+\sqrt{3}\chi_{-1/2}\right\rangle\right]$$

$$= \frac{1}{8}\left[\sqrt{3}\hbar + \sqrt{3}\hbar\right]$$

$$= \frac{\sqrt{3}\hbar}{4}$$

28. For given function $n = 2, l = 1, m = 0$

Number of nodes = $n - l - 1$

$$= 2 - 1 - 1 = 0$$

29. (1) $\frac{p_x^2}{2m} + \frac{m\omega^2 x^2}{2} - qEx$ which is equivalent to

$$\frac{p_x^2}{2m} + \frac{m\omega^2}{2}\left(x^2 - \frac{2qEx}{m\omega^2} + \left(\frac{qE}{m\omega^2}\right)^2 - \left(\frac{qE}{m\omega^2}\right)^2\right)$$

$$\frac{p_x^2}{2m} + \frac{m\omega^2}{2}\left(x - \left(\frac{qE}{m\omega^2}\right)\right)^2 - \frac{qE^2}{2m\omega^2}$$

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega - \frac{q^2E^2}{2m^2\omega^2}$$

30. For attractive delta function wave function is

$$\phi(x) = \sqrt{\lambda}e^{+\lambda x} x < 0$$

$$= \sqrt{\lambda}e^{-\lambda x} x > 0$$

And energy eigen value is

$$= -\frac{m\lambda^2}{2\hbar^2}$$

Which is only one bound state

31.(3) 32.(1) 33.(3) 34.(3) 35.(3) 36.(2) 37.(2) 38.(4) 39.(4) 40.(1)

41.(1) 42.(2) 43.(1) 44.(1) 45.(1) 46.(3) 47.(4) 48.(3) 49.(1) 50. (3)