UGC POINT ACADEMY

LEADING INSTITUE FOR CSIR-JRF/NET, GATE & JAM

BOOKLET CODE

A

PHYSICAL SCIENCE

SUBJECT CODE

05

TEST SEREIS # 4

Mathematical Physics + Thermal+
Statistical + Solid State Physics

Date: 3/12/2015

Timing: 2:00 H
Maximum Marks: 100

INSTRUCTIONS

1. This test paper has a total of 50 questions carrying 100 marks. All sections are compulsory. Question in each section are different type.
2. Read the Questions carefully and mark your appropriate response to the **OMR** sheet.
3. There is Negative marking of **1/4th** for each wrong answer.
4. Mark the response by **Black** or **Blue** Ball Pen only.
5. Any other belongings like Book/ Notes / Electronic device etc are not permitted in the examination hall.
6. Submit your answer sheet (OMR Sheet) to the invigilator before leaving the examination hall.

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1. For the matrix is A as given below, which of them satisfy $A^6 = I$?

(1) $A = \begin{pmatrix} \cos\frac{\pi}{4} & \sin\frac{\pi}{4} & 0 \\ -\sin\frac{\pi}{4} & \cos\frac{\pi}{4} & 0 \\ 0 & 0 & 1 \end{pmatrix}$ (2) $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\frac{\pi}{3} & \sin\frac{\pi}{3} \\ 0 & -\sin\frac{\pi}{3} & \cos\frac{\pi}{3} \end{pmatrix}$

(3) $A = \begin{pmatrix} \cos\frac{\pi}{6} & 0 & \sin\frac{\pi}{6} \\ 0 & 1 & 0 \\ -\sin\frac{\pi}{6} & 0 & \cos\frac{\pi}{6} \end{pmatrix}$ (4) $A = \begin{pmatrix} \cos\frac{\pi}{2} & \sin\frac{\pi}{2} & 0 \\ -\sin\frac{\pi}{2} & \cos\frac{\pi}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$

2. Let J denote a 101×101 matrix with all entries equal to 1 and let I denote the identity of order 101. Then the determinant of $J - I$ is

(1) 0 (2) 1 (3) 0 (4) 100

3. Let $y_1(x)$ and $y_2(x)$ form a complete set of solutions to the differential equation $y''-2xy'+\sin(e^{2^{x^2}})y = 0$, $x \in [0,1]$ with $y_1(0) = 0$, $y_1'(0) = 1$, $y_2(0) = 1$, $y_2'(0) = 1$. Then the Wronkiasn

$W(x)$ of $y_1(x)$ and $y_2(x)$ at $x = 1$ is

(1) e^2 (2) $-e$ (3) $-e^2$ (4) e

4. Consider the function $f(x) = \sqrt{2+x}$ for $x \geq -2$ and the iteration $x_{n+1} = f(x_n) : n \geq 0$ for $x_0 = 1$. What are the possible limits of the iteration?

(1) $\sqrt{2+\sqrt{2+\sqrt{2}+...}}$ (2) –1

(3) 2 (4) 1

5. For a fixed positive integer $n \geq 3$, let A be the $n\times n$ matrix defined $A = I - \frac{1}{n}J$, where I is the identity matrix J is the $n\times n$ matrix with all entries equal to 1. Which of the following statements is NOT true?

(1) $A^k = A$ for every positive integer k (2) Trace (A) = n – 1

(3) Rank (A) + Rank (I – A) = n (4) A is invertible

6. Let A be a 5×4 matrix with real entries such that AX = 0 iff X = 0 where X is a 4×1 vector an 0 is null vector. Then rank of A is

(1) 4 (2) 5 (3) 2 (4) 1

7. Let y: **R** → **R** satisfy the initial value problem $y'(t) = 1 - y^2(t)$, $t \in \mathbf{R}$, y(0) = 0. Then

(1) $y(t_1) = 0$ for some $t_1 \in \mathbf{R}$ (2) y(t) > 1 for all $t \in \mathbf{R}$

(3) y is strictly increasing in **R** (4) y is increasing in (0,1) and decreasing in (1,∞)

8. Consider the equation of an ideal planar pendulum given by

$$\frac{d^2x}{dt^2} = -\sin x$$

Where x denotes the angle of displacement. For sufficiently small angles of displacement, the solution is given by (where a,b are constant)

(1) x(t) = acosht + bsinht (2) x(t) = a + bt

(3) x(t) = ac + be^{2t} (4) x(t) = accost + bsint

9. A linear transformation T rotates each vector in $\mathbf{R}^2$ clockwise through 90^0. The matrix T relative to the standard ordered basis $\left(\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}\right)$ is

(1) $\begin{bmatrix}0 & -1\\-1 & 0\end{bmatrix}$ (2) $\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$

(3) $\begin{bmatrix}0 & -1\\1 & 0\end{bmatrix}$ (4) $\begin{bmatrix}0 & 1\\-1 & 0\end{bmatrix}$

10. Consider the power series $\sum_{n=1}^{\infty} z^{n!}$. The radius of convergence of the series is

(1) 0 (2) ∞

(3) 1 (4) real number greater than 1

11. Let $y_1(x)$ and $y_2(x)$ be the solutions of the differential equation $\frac{dy}{dx} = y + 17$ with initial conditions $y_1(0) = 0$, $y_2(0) = 1$. Then

(1) y_1 and y_2 will never interest (2) y_1 and y_2 will intersect at x = 17

(3) y_1 and y_2 will never intersect at x = e (4) y_1 and y_2 will intersect at x = 1

12. Using the fact that

$$\sum_{1}^{\infty}\frac{1}{n^2}=\frac{\pi^2}{6} \text{ then} \sum_{1}^{\infty}\frac{1}{(2n+1)^2} \text{ equals}$$

(1) $\frac{n^2}{12}$ (2) $\frac{\pi^2}{12}-1$ (3) $\frac{\pi^2}{8}$ (4) $\frac{\pi^2}{8}-1$

13. The power series $\sum_{n=0}^{\infty}3^{-n}(z-1)^{2n}$ converges if

(1) $|z|\le 3$ (2) $|z|<\sqrt{3}$

(3) $|z-1|<\sqrt{3}$ (4) $|z-1|\le\sqrt{3}$

14. Suppose the matrix

$$A=\begin{pmatrix}40 & -29 & -11\\ -18 & 30 & -12\\ 26 & 24 & -50\end{pmatrix}$$

has a certain complex number $\lambda \neq 0$ as an eigenvalues. Which of the following number must also be an eigenvalues of A?

(1) $\lambda + 20$ (2) $\lambda - 20$ (3) 20 (4) –20

15. Let A be a 2×2 non-zero matrix with entries C such that $A^2 = 0$. Which of the following statements must be true?

(1) PAP^{-1} is diagonal for some invertible 2×2 matrix P with entries in R

(2) A has two distinct eigenvalues in C

(3) A has only one eigenvalues in C with multiplicity 2

(4) $Av = v$ for some $v \in C^2$, $v \neq 0$

16. $f(x)$ is a periodic function of x with a period of 2π. In the interval $-\pi<x<\pi$, $f(x)$ is given by

$$f(x)=\begin{cases}0 & -\pi<x<0\\ \sin x, & 0<x<\pi\end{cases}$$

In the expansion of $f(x)$ as a Fourier series of sine and cosine functions, the coefficient of $\cos(2x)$ is

(1) $\frac{2}{3\pi}$ (2) $\frac{1}{\pi}$ (3) 0 (4) $-\frac{2}{3\pi}$

17. The equation of s surface of revolution is

$$z=\pm\sqrt{\frac{3}{2}x^2+\frac{3}{2}y^2}$$

The unit normal to the surface at the point $A\left(\sqrt{\frac{2}{3}},0,1\right)$ is

(1) $\sqrt{\frac{3}{5}}\hat{i}+\frac{2}{\sqrt{10}}\hat{k}$ (2) $\sqrt{\frac{3}{5}}\hat{i}-\frac{2}{\sqrt{10}}\hat{k}$

(3) $\sqrt{\frac{3}{5}}\hat{i}+\frac{2}{\sqrt{5}}\hat{k}$ (4) $\sqrt{\frac{3}{10}}\hat{i}+\frac{2}{\sqrt{10}}\hat{j}$

18. The boundary value problem

$$\frac{d^2y}{dx^2}=y,\ y(0)=0,\ \ y(\infty)=0$$

(1) Has no solution
(2) has many possible solutions
(3) Has a unique solution that is independent of x
(4) Has the unique solution $e^{-x}-e^{x}$

19. Consider the product $(1+z)(1+z^3)(1+z^6)......(1+z^{3m})$. The derivative of this function at $z=0$ is:

(1) 0 (2) 1 (3) m (4) $3m$

20. Given the three matrices

$$\sigma_1=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix},\ \sigma_2=\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix},\ \sigma_3=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

and $[\sigma_i,\sigma_j]\equiv\sigma_i\sigma_j-\sigma_j\sigma_i$, then

$[\sigma_1,[\sigma_2,\sigma_3]]+[\sigma_2,[\sigma_3,\sigma_1]]+[\sigma_3,[\sigma_1,\sigma_2]]$ is

(1) $\sigma_1^2+\sigma_2^2+\sigma_3^2$ (2) $\sigma_1+\sigma_2+\sigma_3$

(3) 0 (4) Identity

21. Let P be a $n\times n$ diagonalizable matrix which satisfies the equations

$P^2=P,\ Tr(P)=n-1$

Det(P) is

(1) n (2) 0 (3) 1 (4) $n-1$

22. A point particle is moving in the (x,y) plane on trajectory given in polar coordinates by the equation $r\sin\left(\theta+\frac{\pi}{4}\right)=5$. The trajectory of the particle is

(1) a parabola (2) a straight line

(3) a circle (4) a hyperbola

23. Consider a forced harmonic oscillator which obey the differential equation,

$$\frac{d^2y}{dt^2}+y=\sin t$$

Which one of the following is the solution of the differential equation with initial condition $y(0)=0$?

(1) $y(t)=6\sin t$ (2) $y(t)=12\sin t+\frac{t}{2}\cos t$

(3) $y(t)=12\sin t-\frac{t}{2}\cos t$ (4) $y(t)=6\cos t-\frac{t}{2}\cos t$

24. The number of times that a circle of radius π, centred at the origin intersects the curve $y = \tan x$ is

(1) 2 (2) 4 (3) 6 (4) 8

25. An unbiased dice thrown five times successively. The number of dots on uppermost surface add up to 28 is

(1) $\frac{16}{7776}$ (2) $\frac{8}{7776}$ (3) $\frac{17}{7776}$ (4) $\frac{10}{7776}$

26. A Taylor series expension of $f(x) = \sin e^x$ about $x = \log \pi / 2$ is

(1) $f(x) = 1 + \frac{\pi^2}{8}\left[x - \ln(\pi/2)^2\right] + ...$

(2) $f(x) = 1 - \frac{\pi^2}{8}\left[x - \ln(\pi/2)\right]^2 + ...$

(3) $f(x) = -\frac{\pi^2}{8}\left[x - \ln(\pi/2)\right]^2 + ...$

(4) $f(x) = -\frac{\pi^2}{8}\left[x + \ln(\pi/2)\right]^2 + ...$

27. The equation of the plane that is tangent to the surface $z = x^2 + y^2$ at point $P(1,2,3)$

(1) $x + y - z = 7$ (2) $2x + 4y - z = 7$

(3) $4x + 2y - z = 7$ (4) $x + 2y + z = 7$

28. Two independent variables m and n which can take the integer values 0,1,2 and so on follow the poission distribution with same mean values μ then

(1) The probabilities of random variables $l = m + n$ is bionomial distribution

(2) The probabilities distribution function of random variable $r = m - n$ is also a poission distribution

(3) The varience of random variable $l = m + n$ is equal to 2μ

(4) The mean value of the random variable $l = m + n$ is μ

29. A 2×2 matrix A has eigenvalue $e^{i\pi/5}$ and $e^{i\pi/6}$. The smallest value of n such that $A^n = I$ is

(1) 20 (2) 30 (3) 60 (4) 120.

30. The function $f(x)$ obeys the differential equation $\frac{d^2 f}{dx^2} - (3 - 4i) f = 0$ and satisfies the conditions $f(0) = 1$ and $f(x) \to 0$ as $x \to \infty$. The value of $f(\pi)$ is

(1) $e^{2\pi}$ (2) $e^{-2\pi}$ (3) $-e^{-2\pi}$ (4) $-e^{2\pi i}$

31. A thermally insulated box of volumes 2V has a partition which divides it into two chambers of equal volume each. Chamber contains He gas at temperature T and pressure P. on removing the partition, the gas molecules in the two chambers mix with each other. The change in entropy of the system is

(1) $-\frac{PV}{T}\ln 2$ (2) 0 (3) $\frac{PV}{T}\ln 2$ (4) $\frac{2PV}{T}\ln 2$

32. The peaks of emission measured from two different black bodies 1 and 2 are found to be at the frequencies ν_1 and ν_2 respectively, with $\nu_1 = 2\nu_2$. The temperature of body 1 is known to be T_1, then the temperature T_2 of body 2 is

(1) $\frac{T_1}{2}$ (2) T_1 (3) $\sqrt{2}T_1$ (4) $2T_1$

33. A vessel containing equal number of molecules of two monoatomic gases. A and B is at temperature T. Their masses are m and 4m respectively. The root mean square velocity of the atoms in the mixture is

(1) $\frac{3}{4}\sqrt{\frac{8kT}{\pi m}}$ (2) $\sqrt{\frac{8kT}{\pi m}}$ (3) $\frac{3}{4}\sqrt{\frac{16kT}{5\pi m}}$ (4) $\sqrt{\frac{16kT}{5\pi m}}$

34. Two solid blocks, one at temperature T_1 and the other $T_2\left(T_1 > T_2\right)$, with the same temperature independent heat capacity C are put in contact with each other. The change in entropy of the universe after they have equilibrated is

(1) $C\ln\left[\frac{(T_1+T_2)^2}{4T_1T_2}\right]$ (2) $C\ln\left(\frac{T_1}{T_2}\right)$

(3) $C\ln\left(\frac{T_1-T_2}{T_1+T_2}\right)$ (4) C

35. Non-interacting Bosons undergo Bose-Einstein condensation (BEC) when trapped in a three dimensional isotropic simple Harmonic potential. For BEC to occur, the chemical potential must be equal to

(1) $\hbar\omega/2$ (2) $\hbar\omega$ (3) $3\hbar\omega/2$ (4) 0

36. Consider a system of N non-interacting spins, each of which has classical magnetic moment of magnitude μ. The Hamiltonion of this system in an external magnetic field $\vec{H}$ is $H = -\sum_{i=1}^{N}\vec{\mu}_i\vec{H}$, where $\vec{\mu}_i$ is the magnetic moment of the ith spin. The magnetization per spin at temperature T is

(1) $\frac{\mu^2 H}{k_B T}$ (2) $\mu\left[\coth\left(\frac{\mu H}{k_B T}\right)-\frac{k_B T}{\mu H}\right]$

(3) $\mu\sinh\left(\frac{\mu H}{k_B T}\right)$ (4) $\mu\tanh\left(\frac{\mu H}{k_B T}\right)$

37. Bose condensation occurs in liquid H_e^4 kept at ambient pressure at 2.17 K. At which temperature will bose condensation occur in H_e^4 in gaseous state, the density of which is 1000 times smaller than that of liquid H_e^4 ?(assume that it is perfect bose gas)

(1) 2.17 mK (2) 21.7 mK (3) 21.7 μK (4) 2.17μK

38. Consider black body radiation contained in a cavity whose walls are at temperature T. The radiation is in equilibrium with the walls of the cavity. The temperature of walls is increased to 2T and the radiation is allowed to come to equilibrium at the new temperature, the entropy of the radiation increases by a factor of

(1) 2 (2) 4 (3) 8 (4) 16

39. A monoatomic gas consists of atoms with two internal energy levels, ground state $E_0 = 0$ and an excited state $E_1 = E$. The specific heat of the gas is given by

(1) $\frac{3}{2}k$ (2) $\frac{E^2 e^{E/kT}}{kT^2\left(1+e^{E/kT}\right)^2}$

(2) $\frac{3}{2}k + \frac{E^2 e^{E/kT}}{kT^2\left(1+e^{E/kT}\right)^2}$ (4) $\frac{3}{2}k - \frac{E^2 e^{E/kT}}{kT^2\left(1+e^{E/kT}\right)^2}$

40. Consider a system of 2N non-interacting spin ½ particles each fixed in position and carrying a magnetic moment μ. The system is immersed in a uniform magnetic field B. The number of spin up particle for which the entropy of the system will be maximum is

(1) 0 (2) N (3) 2N (4) N/2

41. The free energy difference between the super conducting and the normal states of a material is given by $\Delta f = f_S - f_N = \alpha|\Psi|^2 + \frac{\beta}{2}|\Psi|^4$, where Ψ is an order parameter and α and β are constants such that $\alpha > 0$ in the normal and $\alpha < 0$ in the superconducting state, while $\beta > 0$ always. The minimum value of Δf is

(1) $-\alpha^2/\beta$ (2) $-\alpha^2/2\beta$ (3) $\alpha^2/2\beta$ (4) $-5\alpha^2/2\beta$

42. In a band structure calculation, the dispersion relation for electrons is found to be $\varepsilon_k = \beta\left(\cos k_x a + \cos k_y a + \cos k_z a\right)$, where β is a constant and a is lattice constant. The effective mass at the boundary of the first Brillouin zone is

(1) $\frac{2\hbar^2}{5\beta a^2}$ (2) $\frac{4\hbar^2}{5\beta a^2}$ (3) $-\frac{\hbar^2}{\beta a^2}$ (4) $\frac{\hbar^2}{3\beta a^2}$

43. Fermi energy of a certain metal M_1 is $5eV$. A second metal M_2 has an electron density which is 6% higher than that of M. assuming that the free electron theory is valid for both the metal the Fermi energy of M_2 is closest to

(1) 5.6 eV (2) 5.2 eV (3) 4.8 eV (4) 4.4 eV

44. An X-ray beam of wavelength $1.54\,\mathring{A}$ is diffracted from the (1 1 0) planes of a solid with a cubic lattice of lattice constant $3.08\,\mathring{A}$. The first order Bragg diffraction occurs at

(1) $\sin^{-1}\left(\frac{1}{4}\right)$ (2) $\sin^{-1}\left(\frac{1}{2\sqrt{2}}\right)$ (3) $\sin^{-1}\left(\frac{1}{2}\right)$ (4) $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$

45. A metal of atomic weight W and density ρ has FCC structure. The side a of its conventional cubic cell is given by

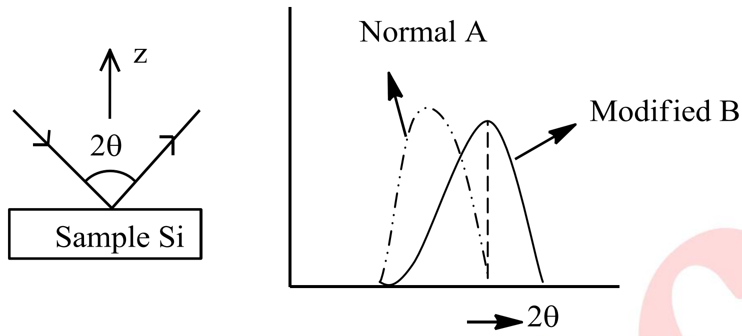
(1) $\left(\frac{W}{\rho}\right)^{1/3}$ (2) $\left(\frac{2W}{\rho}\right)^{1/3}$ (3) $\left(\frac{4W}{\rho}\right)^{1/3}$ (4) $\left(\frac{6W}{\rho}\right)^{1/3}$

46. In an X-ray diffraction experiment on a material with lattice parameter $4.0\,\mathring{A}$, the first peak is obtained at $2\theta = 10^o$. If the same experiment (with the same wavelength of X rays) is performed on a second material with the same crystal structure but a lattice parameter of $2.0\,\mathring{A}$ the first peak will appear at a value of 2θ most nearly equal to

(1) 5^o (2) 10^o (3) 20^o (4) 40^o

47. The figure below shows the Bragg diffraction pat for X-rays of wavelength $1.54\,\mathring{A}$ incident on two crystalline silicon thin film samples A and B. The dated line corresponds to a normal sample A and the continuous line corresponding to another sample B, which is modified due to difference in the growth conditions.

$\lambda_{x\text{-}ray} = 1.54\,\mathring{A}$



These plots suggest that the modified sample B is

(1) Stretched in all directions by 3%

(2) Compressed in all directions by 3%

(3) Stretched in the z direction by 1% and poss compressed in x & y directions

(4) Compressed in the z direction by 1% and poss stretched out in x & y directions

48. Metallic Copper is known to form cubic crystals and the lattice contant is measured from X-ray diffraction studies to be about 0.36 nm. If the specific gravity of copper is 8.96 and its atomic weight is 63.5, one can conclude that

(1) there is insufficient data to distinguish between the options below

(2) the crystals are of simple cubic type

(3) the crystals are a mixture of f.c.c. and b.c.c types

(4) the crystals are of f.c.c. types

49. A lattice is characterized by the following permittivies vectors(in angstroms):
$\vec{a} = 2(\vec{i} + \vec{j}),\ \vec{b} = 2(\hat{j} + \hat{k}), \vec{c} = 2(\hat{k} + \hat{i})$. The reciprocal lattice corresponding to the above is

(1) body centred cubic lattice with cube edge πA^{-1}

(2) body centred cubic lattice with cube edge $2\pi A^{-1}$

(3) face centred cubic lattice with cube edge πA^{-1}

(4) face centred cubic lattice with cube edge $2\pi A^{-1}$

50. Consider the energy E in the first Brillouin zone as a function of the magnitude of the wave vector k for a crystal of lattice constant a. then

(1) the slope of E versus k is proportional to the group velocity

(2) the slope of E versus k has its maximum value at $k = \pi / a$

(3) the plot of E versus k will be parabolic in the interval $(-\pi / a) < k < (\pi / a)$

(4) the slope of E versus k is non-zero for all k the interval $(-\pi / a) < k < (\pi / a)$