21. Find the Poission Bracket of

$$\left[\left(x^2P+px^2\right),x^4\right]$$

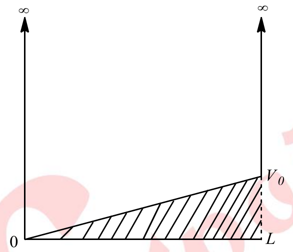
$$=\frac{\partial}{\partial x}\left(x^2P+px^2\right)\frac{\partial}{\partial p}x^4-\frac{\partial}{\partial p}\left(x^2P+px^2\right)\frac{\partial}{\partial x}x^4$$

$$=0-\left(x^2+x^2\right)4x^3\left[xP\right]$$

$$=-8x^5\left(i\hbar\right)$$

Since there is only one pair of xP in the question. If you taken the quantum route it is much longer but you will get the same answer

22. Perturbation $H'=\frac{V_0x}{L}$



$$\Psi_n=\sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L}$$

$$\langle H'\rangle=\int_0^L\Psi_{(n)}^{x}\Psi_n^{(x)}H'dx$$

$$=\int_0^L\left(\frac{2}{L}\sin^2\frac{n\pi x}{L}\right)\frac{V_0x}{L}dx$$ (Doing this integration will require some work)

You just calculate the area of perturbation curve and divided by L

$$\langle H'\rangle=\frac{Area\ of\ shaded\ \Delta}{L}=\frac{1}{2}\frac{LV_0}{L}=\frac{V_0}{2}$$

23. $$I=\int_{-1}^{+1}(1+2x)P_n(x)dx$$

$$=\int_{-1}^{1}P_0(x)P_n(x)dx+2\int_{-1}^{1}P_1(x)P_n(x)dx$$

Use $\int_{-1}^{1}P_n(x)P_m(x)=\frac{2}{2n+1}\delta_{nm}$

$$\therefore\ I=\frac{2}{2n+1}\delta_{n0}+2\left(\frac{2}{2n+1}\right)\delta_{n1}$$

Since $n>1$ $\qquad\delta_{n0}=0\qquad\delta_{n1}=0$

$$\therefore\ \boxed{I=0}$$

24. $\Delta x=\sqrt{\langle x^2\rangle-\langle x\rangle^2}$ and $\Delta p=\sqrt{\langle p^2\rangle-\langle p\rangle^2}$

For 1-D H.O. $\langle x\rangle=\langle p\rangle=0$ for all nth state to calculate $\langle x^2\rangle,\langle p^2\rangle$, you need to known the nth state Eigen function and then do integration but it is long process. So you need an easier route, hence it is

$\langle T\rangle=\langle V\rangle$ .... (1) Viral theorem for 1-D Harmonic Oscillator

$T=K.E$ and $V=P.E,$ E = Total energy

$\langle T\rangle+\langle V\rangle=\langle E\rangle$

Now from (1) $\langle T\rangle=\langle V\rangle$

$$\langle T\rangle+\langle T\rangle=\langle E\rangle$$

$$\langle T\rangle=\frac{\langle E\rangle}{2}$$

$$\frac{\langle p^2\rangle}{2m}=\frac{1}{2}(n+1/2)\hbar\omega$$

$$\langle p^2\rangle=(n+1/2)\hbar\omega(m)$$

$$\langle p^2\rangle=\left(n+\frac{1}{2}\right)\hbar(m\omega)$$

$$\langle V\rangle=\frac{1}{2}K\langle x^2\rangle=\frac{1}{2}\left(n+\frac{1}{2}\right)\hbar\omega$$

$$K\langle x^2\rangle=\left(n+\frac{1}{2}\right)\hbar\omega$$

$$\langle x^2\rangle=\left(n+\frac{1}{2}\right)\frac{\hbar\omega}{K}$$ but $K=m\omega^2$

$$=\left(n+\frac{1}{2}\right)\frac{\hbar\omega}{m\omega^2}$$

$$\langle x^2\rangle=\left(n+\frac{1}{2}\right)\frac{\hbar}{m\omega}$$

$$\langle x^2\rangle\langle p^2\rangle=\left(n+\frac{1}{2}\right)\hbar(m\omega)\left(n+\frac{1}{2}\right)\frac{\hbar}{m\omega}=\left(n+\frac{1}{2}\right)^2\hbar^2$$

$$\therefore \Delta x.\Delta p=\sqrt{\langle x^2\rangle-\langle x\rangle^2}.\sqrt{\langle p^2\rangle-\langle p\rangle^2}$$

$$=\sqrt{\langle x^2\rangle.\langle p^2\rangle}$$

$$=\sqrt{(n+1/2)^2\hbar^2}$$

$$\Delta x.\Delta p=(n+1/2)\hbar$$

For ground state $\boxed{\Delta x.\Delta p=\hbar/2}$

25. $\Psi(t=0) = a\Psi_1 + b\Psi_2$

$\Psi(t=T) = a\Psi_1 e^{-iE_1T/\hbar} + b\Psi_2 e^{-iE_2T/\hbar}$

$$\langle \Psi(t=0) | \Psi(t=T) \rangle = aa^* \langle \Psi_1 | \Psi_1 \rangle e^{-iE_1T/\hbar} + b^* a be^{-iE_1t/\hbar} \langle \Psi_2 | \Psi_1 \rangle + bb^* \langle \Psi_2 | \Psi_2 \rangle e^{-iE_2T/\hbar} + a^* b \langle \Psi_1 | \Psi_2 \rangle$$

$$aa^* = |a|^2, bb^* = |b|^2, \langle \Psi_1 | \Psi_1 \rangle = \langle \Psi_2 | \Psi_2 \rangle = 1, \langle \Psi_1 | \Psi_2 \rangle = 0$$

$$\langle \Psi(t=0) | \Psi(t=T) \rangle = |a|^2 e^{-iE_1T/\hbar} + |b|^2 e^{-iE_2T/\hbar} = 0$$

$$e^{iT/\hbar(E_2 - E_1)} = \frac{-|b|^2}{|a|^2}$$

Equate Real part

$$\cos\frac{T}{\hbar}(E_2 - E_1) = \frac{-|b|^2}{|a|^2}, \qquad |a^2| + |b|^2 = 1$$

$$\cos\frac{T}{\hbar}(E_2 - E_1) = \frac{|a|^2 - 1}{|a|^2} \qquad \therefore |b^2| = 1 - |a|^2$$

$$T = \frac{\hbar}{E_2 - E_1}\cos^{-1}\left(\frac{|a|^2 - 1}{|a|^2}\right) \qquad \because |a|^2 = 2/3 \text{ (given)}$$

$$T = \frac{\hbar}{E_2 - E_1}\cos^{-1}\left(-\frac{1}{2}\right)$$

$$= \frac{\hbar}{E_2 - E_1}\frac{2\pi}{3}$$

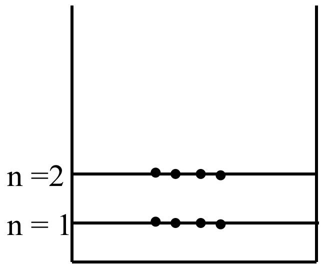
$$\boxed{T = \frac{2\pi}{3}\frac{\hbar}{E_2 - E_1}}$$

26. Answer (3)

Particle in each state will be = (2s+1)

$$= \left(2 \times \frac{3}{2} + 1\right)$$

= 4



Minimum Energy = $4 \times \frac{\pi^2\hbar^2}{2ma^2} + \frac{4 \times 2^2 \pi^2\hbar^2}{2ma^2}$

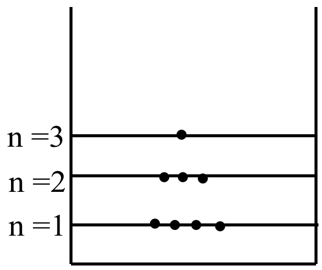
$$= \frac{4\pi^2\hbar^2}{2ma^2} + \frac{16\pi^2\hbar^2}{2ma^2}$$

$$= \frac{20\pi^2\hbar^2}{2ma^2}$$

First excited energy = $\frac{4\pi^2\hbar^2}{2ma^2} + \frac{3 \times 2^2 \pi^2\hbar^2}{2ma^2} + \frac{3^2 \pi^2\hbar^2}{2ma^2}$

$$= \frac{4\pi^2\hbar^2}{2ma^2} + \frac{12\pi^2\hbar^2}{2ma^2} + \frac{9\pi^2\hbar^2}{2ma^2}$$

$$= \frac{25\pi^2\hbar^2}{2ma^2}$$



$\therefore$ Minimum excitation energy = $\dfrac{25\pi^2\hbar^2}{2ma^2} - \dfrac{20\pi^2\hbar^2}{2ma^2}$

$$= \frac{5\pi^2\hbar^2}{2ma^2}$$

$$= 5\left(\text{In unit of } \frac{\pi^2\hbar^2}{2ma^2} \right)$$

27. $S_x = \dfrac{\hbar}{2}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad S_y = \dfrac{\hbar}{2}\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \quad S_z = \dfrac{\hbar}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$\chi = N\begin{bmatrix} 4i \\ 3 \end{bmatrix}$$

$$\chi^* \chi = 1$$

$$N^2\left(-4i \quad 3\right)\begin{pmatrix} 4i \\ 3 \end{pmatrix} = 1$$

$$N^2\left(16+9\right) = 1$$

$$25N^2 = 1$$

$$N = 1/5$$

$$\langle S_x \rangle = N\left(-4i \quad 3\right)\frac{\hbar}{2}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} N\begin{pmatrix} 4i \\ 3 \end{pmatrix} = \frac{N^2\hbar}{2}\left(-4i \quad 3\right)\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 4i \\ 3 \end{bmatrix}$$

$$= \frac{N^2\hbar}{2}\left(-4i \quad 3\right)\begin{pmatrix} 3 \\ 4i \end{pmatrix}$$

$$\langle S_x \rangle = \frac{N^2\hbar}{2}\left(-12i + 12i\right) = 0$$

$$\langle S_y \rangle = \frac{N^2\hbar}{2}\left(-4i \quad 3\right)\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}\begin{pmatrix} 4i \\ 3 \end{pmatrix} = -12\hbar N^2$$

$\langle S_z \rangle = 7/2N^2\hbar$ and $\langle S_x^2 \rangle = \langle S_y^2 \rangle = \langle S_z^2 \rangle = \dfrac{\hbar^2}{4}$

$$\therefore \Delta S_x = \sqrt{\langle S_x^2 \rangle - \langle S_x \rangle^2} = \sqrt{\hbar^2/4 - 0} = \frac{\hbar}{2} = 0.5\hbar$$

$$\Delta S_y = \sqrt{\langle S_y^2 \rangle - \langle S_y \rangle^2} = \sqrt{\frac{\hbar^2}{4} - \left(144N^4\hbar^2\right)} = \hbar\sqrt{\frac{1}{4} - \frac{144}{625}}$$

$$\Delta S_y = 0.14\hbar$$

$$\Delta S_z = \sqrt{\langle S_z^2 \rangle - \langle S_z \rangle^2} = \sqrt{\frac{\hbar^2}{4} - \frac{49}{4}\frac{\hbar^2}{625}} = \hbar\sqrt{\frac{1}{4} - \frac{49}{4 \times 625}} \Rightarrow 0.48\hbar$$

$$\therefore \Delta S_x + \Delta S_y + \Delta S_z = 0.5\hbar + 0.14\hbar + 0.48\hbar$$

$$\Delta S_x + \Delta S_y + \Delta S_z = 1.12\hbar$$

28. Answer (2)

Let E_1 and E_2 are energies of particle B and massless particle

$$E_1 + E_2 = m_A c^2 \qquad \text{................(1)}$$

According to law of conservation of linear momentum, the momentum of particle B and massless particles are equal in magnitude and opposite in sign. Let their momentum be p.

$$E_1^2 = m_B^2 c^4 + p^2 c^2$$

$$E_2^2 = p^2 c^2$$

$$E_1^2 - E_2^2 = m_B^2 c^4 \qquad \text{................(2)}$$

Dividing (2) by (1)

$$E_1 - E_2 = \frac{m_B^2 c^2}{m_A}$$

Subtracting (3) from (1)

$$2E_2 = m_A c^2 - \frac{m_B^2 c^2}{m_A}$$

$$E_2 = \frac{\left(m_A^2 - m_B^2\right)c^2}{2m_A}$$

Momentum of particle B in rest frame of particle A is same as that of massless particle

$$p = \frac{E_2}{c} = \frac{\left(m_A^2 - m_B^2\right)c}{2m_A}$$

29. Answer (4)

In ground state the orbital angular momentum is zero. The electron has only spin angular momentum. Energy of interaction of electron with magnetic field is given by

$$E = -\bar{\mu}, \bar{B} \quad = -\frac{e}{m}\vec{S}.\vec{B} \quad = -\frac{e}{m}S, B \quad = -\frac{e}{m}m,\hbar B \quad = -m, B\frac{e\hbar}{m}$$

M, has two values +1/2 and -1/2

The interactior with magnetic field produces splitting of $B\frac{e\hbar}{m}$. So it will absorb radiation of wavelength which will correspond to this energy difference

$$\frac{hc}{\lambda} = B\frac{e\hbar}{m}$$

$$\lambda = \frac{2\pi mc}{eB}$$

$$= \frac{9.1\times10^{-31}\times3\times10^{8}\times2\times3.14}{1.6\times10^{-19}\times10}$$

$$= 1.07mm$$

30. Answer (1)

$\psi_0 = Ae^{-a\sqrt{k}x^2}$ where 'A' is normalisation constant

When parameter k is suddenly changed to 4k, the wavefunction associated with ground state of new potential is given by

$\psi' = A'e^{-2a\sqrt{k}x^2}$

Probability that particle will be found in ground state

$p = \left|\langle \psi' | \psi \rangle\right|^2$

$$\langle \psi | \psi \rangle = A^2 \int_{-\infty}^{\infty} e^{-2a\sqrt{k}x^2} dx = 2A^2 \int_0^{\infty} e^{-2a\sqrt{k}x^2} dx$$

Put $a\sqrt{k}x^2 = y$

$$x = \frac{y^{1/2}}{a^{1/2}k^{1/4}}$$

$$dx = \frac{1}{a^{1/2}k^{1/4}} \frac{1}{2\sqrt{y}} dy$$

$$\langle \psi | \psi \rangle = 2A^2 \frac{1}{2a^{1/2}k^{1/4}} \int_0^{\infty} y^{-1/2} dy$$

$$= \frac{A^2}{a^{1/2}k^{1/4}} \overline{|1/2} = \frac{A^2\sqrt{\pi}}{a^{1/2}k^{1/4}}$$

From Normalisation equation

$\langle \psi | \psi \rangle = 1$

$$\Rightarrow A = \frac{a^{1/4}k^{1/8}}{\pi^{1/4}}$$

$$\psi = \frac{a^{1/4}k^{1/8}}{\pi^{1/4}} e^{-a\sqrt{k}x^2}$$

$$\psi' = \frac{a^{1/4}(4k)^{1/8}}{\pi^{1/4}} e^{-2a\sqrt{k}x^2}$$

$$\langle \psi' | \psi \rangle = \frac{a^{1/2}k^{1/4}4^{1/8}}{\pi^{1/2}} \int_{-\infty}^{\infty} e^{-3a\sqrt{k}x^2} dx$$

$$= \frac{2a^{1/2}k^{1/4}2^{1/4}}{\pi^{1/2}} \int_0^{\infty} e^{-3a\sqrt{k}x^2} dx$$

Let $3a\sqrt{k}x^2 = y$

$$x = \frac{1}{3^{1/2}a^{1/2}k^{1/4}} y^{1/2}$$

$$dx = \frac{1}{3^{1/2}a^{1/2}k^{1/4}} \frac{1}{2.\sqrt{y}} dy$$

$$\langle \psi' | \psi \rangle = \frac{2a^{1/2}k^{1/4}2^{1/4}}{\pi^{1/2}} . \frac{1}{3^{1/2}a^{1/2}k^{1/4}} \frac{1}{2} \int_0^{\infty} y^{-1/2} e^{-y} dy$$

$$= \frac{2^{1/4}}{3^{1/2}\pi^{1/2}} \overline{|1/2} = \frac{2^{1/4}}{3^{1/2}}$$

Probability $P = \left|\psi'|\psi\right|^2 = \frac{\sqrt{2}}{3} = 0.47$

36. Answer(2)

The surface belongs to family of level surface given by

$S : \cos x \cosh y - z = 0$

Normal to the surface is given by

$$\hat{n} = \frac{\nabla S}{|\nabla S|}$$

$\nabla S = -\sin x \cosh y\, \hat{i} + \cos x \sinh y\, \hat{j} - \hat{k}$

At $x = \frac{\pi}{2}, y = 0$

$$\nabla S = -\hat{i} - \hat{k}$$

$$\hat{n} = \frac{\nabla S}{|\nabla S|} = \frac{-\hat{i} - \hat{k}}{\sqrt{2}}$$

$\hat{n}$ lies in xz plane

59. $\nabla.E = \frac{\rho}{\epsilon_0}$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 E_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta E_\theta\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}E_\phi$$

$$= \frac{A}{r^2} + \frac{3\sin^2\phi\cos\phi}{r\sin\theta} = \frac{\rho}{\epsilon_0}$$

60. $B = \mu_0 I(t)\hat{z}$

$B = \mu_0 n I_0 t\hat{z}$

$$E 2\pi r = \pi r^2 \frac{\partial B}{\partial t} = \pi r^2 \mu_0 n I_0$$

$$E = \frac{\mu_0 n I_0}{2} r\hat{\phi}$$

Momentum density = $\frac{s}{c^2}$

$$= \frac{-\mu_0 (nI_0)^2}{2c^2} rt\,\hat{r}$$

61. $B = \frac{E_0}{c}\sin(Kx - wt)\hat{z}$

$$S = \frac{\vec{E} \times \vec{B}}{\mu_0} = \frac{E_0^2}{c\mu_0}\sin^2(Kx - wt)\hat{x}$$

$$\langle S \rangle = \frac{E_0^2}{2c\mu_0}$$